The internal efficiency of Index Option Markets:
Tests on the Italian Market

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Abstract

The aim of the present paper is to provide evidence on the internal market efficiency of the Italian index option market. To this end a model-free approach is taken, whereby strategies involving only options are tested by means of a high frequency dataset covering the period 1 September – 31 December 2002. This piece of research thus completes our previous analysis (Brunetti and Torricelli(2003, 2006)), which focused on the cross-market efficiency of the same market. The results obtained further support the efficiency of one of the most important index options markets in Europe.

Keywords: index options, internal market efficiency, no-arbitrage, option spreads.

JEL Classification: G13, G14

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1. Introduction

The efficiency of a financial market is of greatest importance for its functioning and its development and can be investigated either by means of model-based tests or by exploring arbitrage pricing relationships that must hold among financial assets. Given that the former approach involves a joint test of the market efficiency and of the option pricing model specification, most empirical research rests on the definition of market efficiency as the absence of arbitrage opportunities.¹

When option market efficiency is investigated, two are the relevant notions of efficiency: the cross markets efficiency, which is based on tests of the joint efficiency of the option and the underlying market, and the internal option market efficiency, that aims at assessing the existence of arbitrage opportunities within the very same option market. The former tests of efficiency are performed on the lower boundary conditions that have to hold for call and put options and on the most famous arbitrage pricing relationship, i.e. the put-call parity. By contrast, the latter tests of efficiency are performed on various types of arbitrage strategies involving options only, such as box and butterfly spreads. Ackert and Tian(2001) stress that “As only options are involved, an examination of these relationships may provide a superior test of parity among index options”.

Since the seminal paper by Stoll (1969), most of the literature on the efficiency of index options has focused on US markets (e.g. Ackert and Tian(2001), Evnine and Rudd(1985), Kamara and Miller(1995)), while only a few contributions have investigated some relatively new European index option markets. As far as we know, only a few recent papers, propose efficiency tests on European markets and specifically: Capelle-Blancard and Chaudhury (2001) for the French index (CAC40) option market, Mittnik and Rieken(2000a,b) for the German index (DAX) option, Cavallo and Mammola (2000) and Brunetti and Torricelli (2003, 2006) for Italian index (MIB30) option market. ² Moreover, but for the French study, the existing analyses on European markets focus on cross-

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¹ In line with most of the literature cited in this paper, this is the notion of efficiency we adopt here. However, it should be stressed that, even when the no-arbitrage restrictions hold, the market may still be inefficient in other respects: for example, market prices might still deviate from theoretical (e.g. Black and Scholes) prices.

market efficiency only. This is true also for the two mentioned studies on the Italian market, i.e. Brunetti and Torricelli (2003, 2006) and Cavallo and Mammola (2000).

The aim of the present paper is to provide evidence on the internal market efficiency of the Italian Index Option (Mibo) market thus adding evidence to the existing studies and completing our previous analysis. To this end and for comparability with other papers (mainly Capelle-Blancard and Chaudhury (2001) and Ackert and Tian(1998, 2001)), we will test the call (put) spread, the call (put) butterfly spread and the box spreads. This piece of research allows for a thorough comparison with the internal market efficiency analyses conducted on other markets, and specifically with the French one, which represent one of the most important index option markets in continental Europe. Therefore, this type of comparative analysis is also important in the light of the issue of the financial markets integration in Europe.

The paper is organised as follows. Section 2 introduces the Mibo contract and motivates the present study with respect to the Italian market features and the existing studies. In section 3 the arbitrage relationships that must hold within an option market are described. Section 4 illustrates the high frequency dataset used in the present work. The internal market efficiency tests and the results for the Mibo market are presented and discussed in Section 5, while Section 6 presents a comparative discussion with other studies. Last section concludes.

2. The Italian Index Option Market

The Mibo contract, which was introduced in the Italian Derivatives Market (IDEM) in November 1995, is a European-style index option contract based on one of the most representative Italian indexes, the Mib30. Every day, six different expirations are quoted: four quarterly (March, June, September and December) and two monthly (the nearest two months). The expiration day is the third Friday of the expiration month, if the Exchange is open, the previous day of open Exchange otherwise. At expiration, in the money options are automatically exercised. The exercise prices have fixed increments of 500 index points and every day at least nine different strikes for each expiration are

\[\text{3 Since 2}^{\text{nd}} \text{ June 2003 a new index has been quoted on the Italian Market: the S&P/Mib. This index, whose components are not fixed and that at the time of writing contains 40 assets, from September 2004 has}\]
quoted: one at the money, four in and four out of the money. The cash settlement of the options is overseen by the Italian Clearing House, Cassa di Compensazione e Garanzia (CC&G), which also calculates and manages the margins. By now, no limits are provided for open interests and price changes during the negotiation time (9:15-17:40).

From its birth in 1995 up to 2001, the volume of Mibo contracts negotiated has significantly increased and the notional value of the Mibo exchanged every year is very important and even bigger than that of the Isoα, i.e. Italian option contracts on single stocks (see Figures 1, 2 and 3). Thus, despite the IDEM is a relatively young market, it has become the fifth derivatives market in Europe (after Liffe, DTB, Monep and Dutch Eurex).

By inspection of Figures 2 and 3, a decline both in volumes and notional values is observable as from 2001. The latter, which is stronger, is quite natural since it follows the decline in the underlying index Mib30. By contrast, the dynamics of the traded contracts represented in Figure 1 does not follow the S shape of a logistic growth curve typically characterizing the life cycle of a derivative. Of course, the life cycle of derivatives is not necessarily uniform. As Remolona(1993) puts it “Demand factors have shaped and stretched the various S curves to cause some contracts to grow much faster and others much slower than might be indicated by a simple life cycle explanation.”

However, in the case of the Mibo life cycle, we observe a concave shape and a natural question arises: is the drop following the year 2000 justified by the demand conditions or is it related to changes in the market efficiency? An answer to this question was provided in Brunetti and Torricelli (2003, 2006) based on the investigation of the cross-market efficiency, which supported a high level of efficiency of the Italian index option market in the period under analysis. However, those studies – as well as Cavallo and Mammola (2000) - are mainly based on tests of the put-call parity, which may not indicate market inefficiency. In fact, arbitrage at low cost may be difficult to implement due to the specific nature of the underlying, which requires either portfolio replication or the use of

become the new underlying of the Italian index derivatives. However, contract specifications have remained practically unchanged (see www.borsaitalia.it).

4 Specifically, the years 1999 and 2000 registered an increase in trading volumes of 38% and 27% respectively, the drop in the years 2001 and 2002 amounted to 4% and 5% respectively.
futures contracts. For example Ackert and Tian (1998) analyse the Canadian index and option market and conclude that while the option market efficiency increased in the period under investigation the connection between option and underlying market did not. Based on these arguments, the present paper aims to complement previous studies on the efficiency of the Mibo market, which are based on arbitrage pricing relationships involving both the option and underlying market. The arbitrage relationships used in the present paper are discussed in the next section.

3. Tests of the internal market efficiency

Only small body of papers in the literature offers an analysis of the internal option market efficiency beside the cross-market one (e.g. Billingsley and Chance (1985), Chance (1986), Ronn and Ronn (1989), Capelle-Blancard and Chaudhury (2001) and Ackert and Tian (1998, 2001)). Moreover, most of them focus on the US market, the only exceptions being Capelle-Blancard and Chaudhury (2001) and Ackert and Tian (1998) who analyse the French and the Canadian Market respectively. As for the methodology, this literature is based on a model-free approach, which implies efficiency tests performed on arbitrage strategies involving options only. Many are the advantages of such an approach: it does not require specification of any pricing model and hence does not rest on the validity of the model underlying assumptions, it takes market frictions into account and it considers only feasible transactions. Moreover, in contrast to cross-market efficiency tests, strategies involving options only are not affected by different closing times in the stock and option markets and do not involve the problems connected with the replication of the underlying. On the other hand, these tests require high data synchronicity, which is attainable when a high frequency dataset is available.

More specifically, the most common strategies involving options only are: call and put spreads, call and put butterfly spreads and box spreads.

The call (put) spread can be created by buying a call (put) option with a certain strike price and selling a call (put) option on the same underlying with a different strike price. Depending on whether this latter price is higher or lower than the strike of the option purchased the spread is referred to as a bull or a bear spread.
The call (put) butterfly spread involve positions in options with three different strike prices. It can be created by buying a call (put) option with a relatively low strike price, buying a call option with a relatively high strike price and selling two call (put) options with a strike price halfway between the previous two.

The box spread can be created by combining a bull call spread and a bear put spread. Ackert and Tian (2001) stress that the box spread is similar to the put-call parity “except that two pairs of matched call and put options are used and the index itself is removed from the relationship”.

In an efficient market all the options strategies just described produce a positive payoff in any state of the world.

In order to formally represent the payoffs of these strategies, let:

\[ C_i^a/P_i^a \] = i-th call/put ask price, \( i=1,2,3 \);

\[ C_i^b/P_i^b \] = i-th call/put bid price, \( i=1,2,3 \);

\( K_i \) = strike of the i-th option, \( i=1,2,3 \);

\( r \) = risk-free rate;

\( t \) = time to maturity;

\( TC_{c,p} \) = call/put transaction costs;

and define:

\[ w = \left( \frac{K_3 - K_2}{K_3 - K_1} \right) \]

The payoffs of call and put spreads are represented by the following relationships respectively:

\[ (C_2^a - C_1^b) + (K_2 - K_1)\exp(-rt) + 2TC_c \geq 0 \]  \hspace{1cm} (1)

\[ (P_1^a - P_2^b) + (K_2 - K_1)\exp(-rt) + 2TC_p \geq 0 \]  \hspace{1cm} (2)

\[ ^5 \text{As Ackert And Tian (2001) put it “As only options are involved, an examination of these relationships may provide a superior test of parity among index option”}. \]
The payoffs of call and put butterfly spreads are represented by the following relationships respectively:

\[ wC_1^a + (1 - w)C_3^a - C_2^b + 3TC_c \geq 0 \]  
\[ wP_1^a + (1 - w)P_3^a - P_2^b + 3TC_p \geq 0 \]  

The payoffs of box spreads are represented by the following relationships:

\[ (C_1^a - C_2^b) + (P_1^b - P_3^a) + (K_1 - K_2) \exp(-rT) + 2TC_c + 2TC_p \geq 0 \]  
\[ (C_2^a - C_1^b) + (P_2^b - P_1^a) + (K_2 - K_1) \exp(-rT) + 2TC_c + 2TC_p \geq 0 \]

4. The Dataset

The dataset used in the present analysis covers the period 1st of September – 31st December 2001 and was kindly provided by Borsa Italiana Spa. More precisely, the Mibo dataset includes, for each option transaction: the negotiation hour, the clearing hour, the type, the maturity, the option price (expressed in index points, each worth 2.5 €) and the quantity of options traded.

The no arbitrage relationships (1) – (6) hold for couples (or, as for the call/put butterfly spreads, triples) of options with identical maturity and instant of trading but with different strikes. Thus, some filters have to be applied to the original data set. More precisely, as for the option prices synchronicity, the intra-day high frequency data set allows to impose a very high level of price synchronicity. Specifically, following Capelle-Blancard and Chaudhury (2001), we retain only those put/call options pairs traded consequently and within 60 seconds, in order to impose all prices in a given arbitrage condition to be within the same minute. This level of synchronization is much

\[ \text{In the following, we will consider only the negotiation hour as an indicator of the time of the exchange, given that the gap between the negotiation and the clearing hour is less than one second in the 99.11% of the cases.} \]
higher than that imposed by Ackert and Tian (1998, 2001) who, in both papers, use daily data.

As for maturity matching, we first remove all couples of options characterised by different expiration date. Then, in order to implement the strategies (1) – (6), we make sure that the exercise prices are not equal.\(^7\)

To perform the empirical analysis, the risk-free rate has to be chosen and the transaction costs have to be determined. As a proxy for the risk-free rate, we use the Euribor 1, 3, 6 and 12 months, consistently with options maturity (source: Datastream). The choice is made both for comparability with other studies (in particular Capelle-Blancard and Chaudhury (2001)) and because alternative choices (e.g. an IRS rate) would not affect results, given that these types of rates are not significantly different in the period under investigation.

Transaction costs, although very difficult to estimate, are of ultimate importance in this kind of empirical tests. Indeed, there are many components that have to be considered (commissions, trading and clearing fees, costs deriving from bid and ask prices, short selling costs etc.) and each of them depends on the kind of strategy, on the size of transactions and on the investors type (e.g. retails vs. arbitrageurs) and tends to vary over time.

Nevertheless, on the Italian market clearing fees are negligible (see also Cavallo and Mammola (2000)) and the same is true for short-selling costs, since that repo and risk-free interests rates are very low and similar. Hence, as far as transaction costs are concerned, we will focus just on commission costs and the costs deriving from the bid-ask option spread.

By inspection of options trade commissions on the IDEM, the Italian option market appears remarkably diversified. Commissions depend on the type of investors as well as on the means of trade: for example, arbitrageurs usually face low commissions because of the high yearly volume of transactions they realize, even though retail investors who implement trading on line can obtain low commissions too. On the basis of this latter

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\(^7\) These filters implies that only a few of the 229070 original observations were kept. More precisely, for the call (put) spread we retain 12.04% (11.59%) of the total original data set, for the call (put) butterfly spread 2.73% (2.67%) and 0.39% for the box spreads, equal to of the original observations. The sensible disparity in the number of observation retained is essentially due to the different number of options involved in the arbitrage relationships: two in the spreads, three in the butterfly spread and four in the box spreads.
observation, we carry out our empirical study of the no arbitrage relationships (1) – (6) assuming four different commissions levels, which we attribute to four different types of traders:

1. MINIMUM, equal to 1 € for option traded, which is intended to represent arbitrageurs who realize yearly high volume of transactions;
2. MEDIUM-LOW, equal to 10 €, which is intended to represent professional investors with low volume of transactions or particularly active retail investors;
3. MEDIUM-HIGH, equal to 25 €, which is intended to represent retail investors who trade options online;
4. HIGH, equal to 40 € for option traded, which is intended to represent retail investors who trade options only occasionally.

As for bid and ask Mibo quotations, since they are not available in our original dataset, they also have to be estimated. To this end, we create a suitable dataset downloading the bid and ask Mibo quotations available on the Finance section of www.yahoo.com in each trading day of open Exchange, from 3 February to 7 March 2003. Then, on the basis of this sample, we estimate the average option bid-ask spread (as suggested by Phillips and Smith (1980)) and we assume it constant over time, as it is common in literature (see also Capelle-Blancard and Chaudhury (2001)). We find that the mean bid (ask) price resulted about 0.923 (1.062) times the trading price. Thus, multiplying the trading prices available in our data set by these values, we get the estimated bid (ask) options quotations.

5. Empirical results

In order to better emphasize the role of market frictions in absorbing most of the arbitrage opportunities, we will present our results under three different scenarios: scenario A, in which we assume a frictionless market; scenario B, in which we include only the costs deriving from the option bid-ask spread and finally scenario C, in which we take into account the bid-ask costs as well as the commission costs.

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8 Even though this period does not correspond with the one under investigation, we can assume that the average bid-ask spread of index option prices has not remarkably changed, given that we assume it constant over time.
Scenario A
In this scenario we test the no arbitrage conditions (1)–(6), whereby the following assumptions are taken:

\[ TC_C = TC_P = 0 \]
\[ C_i^a = C_i^b = C_i \]
\[ P_i^a = P_i^b = P_i \]

with \( C_i \) and \( P_i \) transaction prices.

[Table 1 about here]

Table 1 reports the results obtained in this first scenario. As for spreads and butterfly spreads, the frequency of violations is very low, even under the hypothesis of frictionless market. In fact, over the whole sample we observe only 0.0018% (0.27%) cases of violations for the call (put) spread and 0.34% (0.47%) for the call (put) butterfly spread. On the other hand, the percentages of the violations of the box spreads no-arbitrage conditions are much higher than those reported above for spreads and butterfly spreads. This result is only apparently surprising. In fact, in this first scenario where no commission costs nor bid ask spread are taken into account, the l.h.s. of relationship (6) is just the negative of the l.h.s. of equation (5). As a consequence, the percentages reported in the two final lines of Table 1 inevitably need to sum up to one.

[Table 2 about here]

Table 2 reports the average values (in €) of the arbitrage opportunities for each relationship in this scenario. Some values are considerable (see call spreads and box spreads (b)). However, overall they are not reliable indicators of the potential arbitrage opportunities existing on the Italian Index Option Market, given that they can be eroded by commission costs and/or bid ask spreads.

Scenario B
Tests in this scenario ignore the commission costs but consider the bid ask spread on option prices, which also in literature is referred to as the most important among the
implicit transaction costs (e.g. Demsetz (1968), Phillips and Smith (1980) and Stoll (1989)). More precisely, we impose:

\[ TC_c = TC_p = 0 \]

and, as illustrated in Section 3, we assume the following:

\[ C^a_i = 1.062 \times C_i \quad C^b_i = 0.923 \times C_i \]

\[ P^a_i = 1.062 \times P_i \quad P^b_i = 0.923 \times P_i \]

Table 3 and Table 4 report, respectively, the frequency and the average amount of the violations to the no arbitrage relationships.

A few comments are here in order. As expected, the inclusion of the bid-ask spread significantly reduces the frequency of the violations of the put spread, the butterfly spreads and the box spreads. As for the call spread, the frequency is unchanged but the average arbitrage profit reduces by more than 40%. Moreover, the mean profits of butterfly spreads have also reduced, while the average amount of arbitrage gains stemming from put spreads and box spread has significantly increased. A possible explanation of this rests on the fact that the bid-ask option spreads, which are not considerable, wipe away those arbitrage opportunities associated to small profits, while leaving those associated to considerable profits. Thus an increase in the average profit follows.

Scenario C

By including the costs deriving from both the bid-ask spread and the commission costs, the most realistic scenario can be obtained. As discussed in Section 3, four different commission levels are assumed, i.e.:

\[ TC_c = TC_p = 1, 10, 25, 40 \text{ €} \]

and as in Scenario B the following bid-ask price relationships are assumed:

\[ C^a_i = 1.062 \times C_i \quad C^b_i = 0.923 \times C_i \]

\[ P^a_i = 1.062 \times P_i \quad P^b_i = 0.923 \times P_i \]

Table 5, 6 and 7 report the results obtained in this last scenario for call and put Spreads, butterfly spreads and box spreads respectively.
Overall, the inclusion of commission costs further reduces the frequency of violations with respect to the previous scenario. In particular, as for call butterfly spreads and box spreads (5), arbitrage opportunities completely disappear.

The importance of transaction costs is once again stressed by the results obtained for put butterfly spread. Table 6 in fact shows how the frequency of arbitrage opportunities decreases, as the level of the transaction costs raises. On the other hand, as far as call and put spreads and box spread (6) are concerned, some more cases of violations still persist, even for retail investors. However, the cases of violations are absolutely exceptional: respectively only 5, 2, and 3 cases of arbitrage opportunities recorded over the whole four-month period.

The average deviations from the no-arbitrage conditions (1) – (6), reported in Table 8, indicate significant arbitrage opportunities by means of both call and put spreads and box spread (6). In other words, although very exceptional, the arbitrage opportunities existing on the Italian index option market can be sizeable, even for retail investors.

In order to identify possible common features of the options violating the no-arbitrage conditions, an explorative analysis of the cases of arbitrage opportunities has been conducted. Overall, the most relevant arbitrage opportunities, namely all the 5 cases of violations of the call spread no-arbitrage relationship, involve options with maturity longer than three months. This result points at highlighting some influence of time to maturity on the frequency of violations and yet has to be interpreted with caution, given that only a few violations are detected over the whole sample and hence the analysis is based on a few observations only.

The tests described so far are normally addressed to as ex-post tests. A few authors (e.g. Capelle-Blancard and Chaudhury (2001), Mittnik and Rieken (2000a), Ackert and Tian (2001)) discuss the importance of performing also ex-ante tests, which are essentially meant to check whether the detected arbitrage opportunities persist long enough to be
exploited by an investor. Ex-ante tests are performed as follows: first, an arbitrage opportunity is singled out, then all the available transaction prices within the next predefined execution window\(^9\) for the same group of options are considered. However, Capelle-Blanchard and Chaudhury stress that, when the number of ex-post violations is too small, which is the case in their study and in the present work, the ex-ante tests are less informative. Despite this observation, we have conducted the ex-ante tests and observed that the arbitrage opportunities detected are both very rare and not repeated in a reasonable execution window (up to two weeks). Therefore, none of the violations detected is actually exploitable for the investors.

### 6. Comparison with other studies

As stressed in the Introduction, most of the literature on index option market efficiency focuses on the notion of cross-market efficiency, but for Capelle-Blancard and Chaudhury (2000) and Ackert and Tian (1998, 2001) who perform an analysis of the internal market efficiency too. Therefore, in this section we will compare our results mainly with those reported in the latter two studies. Moreover, it has to be stressed that Capelle-Blancard and Chaudhury (2000) perform their analyses under different transaction costs scenarios, which are very close to ours; furthermore, they use a dataset that is very similar to ours both for data frequency (high frequency intraday prices) and for number of observation in the sample. As a consequence, the French study represents the most comparable work. On the other hand, Ackert and Tian (1998, 2001) investigate US market efficiency in one single scenario and considering one single level of transaction costs.\(^{10}\) For this reason, in the following their results are compared with our results in scenario C (i.e. professional arbitrageurs with minimum level of transaction costs).

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\(^9\) As for the length of the execution window, in literature different choices are made: Capelle-Blancard and Chaudhury (2001) take a fifteen-minute window, while Mittnik and Rieken (2000a) take different lengths ranging from one minute to one day.

\(^{10}\) It should be stressed that the assumption taken by of Ackert and Tian (2001) corresponds their main objective, i.e. to assess whether efficiency of the S&P 500 index option market has been enhanced by the introduction of Standard and Poor’s Depository Receipt (SPDRs), i.e. traded stock baskets which should ease the replication of the underlying index.
Call & Put Spread

Ackert and Tian (2001), using daily closing prices of the S&P 500 Options, find that the call (put) spread no-arbitrage relationship is violated in 205 (0.40) % of cases over the whole decade under analysis (1986 -1996). As for the amount of violations, Ackert and Tian (2001) find for both these relationships very low levels of arbitrage opportunities: only 1.05 $ and 1.30 $ respectively. This means that the profitable arbitrage opportunities on the Italian Index Option Market are definitively more sporadic than on the Us Market, even though their size is on average much more considerable.

On the Cac 40 Index Option Market, Capelle-Blancard and Chaudhury (2000) find, under the hypothesis of a completely frictionless market, a frequency of call (put) spread violations of 0.34% (0.01%), quite similar to our results: respectively 0.018% (0.27%). On the other hand, once the transaction costs are included, Capelle-Blancard and Chaudhury (2000) observe that all the violations present on the market disappear, while in our analysis very few cases (only 5) of violation still persist. In any case, the detected frequencies of violations of the no-arbitrage conditions under analysis are very similar, under all the scenarios considered, thus confirming a strong similarity between the two European markets.

Nevertheless, a difference emerges as far as the average amount of the arbitrage opportunities is concerned: in fact, Capelle-Blancard and Chaudhury (2000) find that, once all transaction costs are taken into account, a retail investor can achieve on average only a 11.84 € performing a call spread strategy and 16.17 €\footnote{Capelle-Blancard and Chaudhury (2000) report their results in index points, which we have transformed in monetary amounts by considering that on the Monep each index point is equal to 1 €.} performing a put spread strategy, while we find that in a similar scenario on the Italian Index Option Market a retail investor can achieve much higher profits (see Table 8).

Call & Put Butterfly Spread

Ackert and Tian (2001) find that the call (put) butterfly spread are violated in 3.08% (0.91%) of the cases, while in our study we detect only 0.016% for put butterfly spread and no violation for call butterfly spread. This confirms again that the arbitrage opportunities are much more frequent on the Us Market than on the Italian Index Option Market.

\footnote{Capelle-Blancard and Chaudhury (2000) report their results in index points, which we have transformed in monetary amounts by considering that on the Monep each index point is equal to 1 €.}
However, Ackert and Tian (2001) find that the average amount of profits attainable performing both these strategies is around 1 $, which is much a smaller value than the one we find for put butterfly spread (26.69 €)\textsuperscript{12}.

In the scenario without transaction costs, Capelle-Blancard and Chaudhury (2000) find 1.34\% (2.47\%) cases of violations of the call (put) butterfly spreads, while, in a similar scenario, we observe even lower frequencies: 0.34\% (0.47\%) over the whole sample. Once transaction costs are taken into account, both the French and the present study find that all the arbitrage opportunities attainable by means of a call butterfly spread are completely swept away. As far as the put butterfly spread is concerned, we obtain better results than Capelle-Blancard and Chaudhury (2000): they report 18 cases of arbitrage opportunities for retail investors (corresponding to 0.06\% of their sample observations), while we observe no case of arbitrage opportunities at all for the same kind of investors. In this scenario we observe only two arbitrage opportunities for professional arbitrageurs and one single case when a medium-low level of transaction costs is accounted for (respectively 0.033\% and 0.016\% of the available observations over the whole sample).

**Box Spread**

In the existing literature, box spreads have been more often empirically analysed than other types of spreads. For example, Billingsley and Chance (1985) investigate the box spreads using daily data on equity American options, Ronn and Ronn (1989) study the profitability of box spreads using CBOE stock options data, Marchand, Lindley and Followill (1994) use S&P500 futures option data to test long box spread, which corresponds to the no-arbitrage relationship (5) in the present study. The overall finding is that arbitrage profits emerge only if transaction costs are excluded or set at very low levels. However, none of the above cited studies is actually useful for a comparison, given that options with different underlying (equity or futures) are used in the empirical analysis.

Thus, we compare our results only with Capelle-Blancard and Chaudhury (2000), Ackert and Tian (2001) and Ackert and Tian (1998), where the efficiency of the Toronto 35 Index Option Market is investigated.

\textsuperscript{12} This is the mean profit when the minimum level of transaction costs is considered; as transaction costs for retail are included no arbitrage opportunity persists, so that the average profit attainable turns out to be zero.
In this latter work, the authors observe, before the introduction of the TIPs (Toronto Index Participation Units, launched in 1990), quite frequent box spreads violations on the Canadian Index Option Market, even when transaction costs are accounted for: 7.39% and 8.40% for no-arbitrage conditions (5) and (6) respectively. However, the mean profits they observe are, on average, under 0.30 $ for each strategy.

As for Ackert and Tian (2001), by taking the transaction costs into account, they find significant violations of box spreads based on S&P 500 Index Options. In particular, they observe that (5) is violated in 21.02% of case, while (6) is violated by 23.78% of all the available observations. The mean profit detected is, also in this case, very low and around 1 $.

In both cases, the violations are much more frequent on the US Index Option Market than on the Italian Market, on which, once the transaction costs are taken into account, only three violations persist; but the average profits attainable making use of these very rare arbitrage opportunities are much higher on the Italian Market (around 340 €).

In the frictionless scenario Capelle-Blancard and Chaudhury (2000) find 43.26% and 56.74% cases of violations of the box spread relationships. However, with modest transaction costs (Scenario 2 in the French study), the box spread violation frequencies drop around 13% and 11% and further drops to around 10% and 8% in Scenario 3. As for retail investors, they do not detect any arbitrage opportunity at all.

In our study, we find very similar results: under the hypothesis of a completely frictionless market, we observe very high frequencies of violations for both the box spreads no arbitrage conditions, while once the transaction costs are taken into account only three profitable arbitrage opportunities still persist. Thus, considering transaction costs, our results indicate a similar high level of internal options market efficiency (in terms of frequency of violation) in the Italian and French index options market that, in both cases, is significantly greater than in the US Market.

If the frequency of violations of no-arbitrage relationships on the Italian Index Option Market is lower than in other studies, the size of the deviations from these conditions is clearly bigger. The average amount of arbitrage opportunities detected by Capelle-Blancard and Chaudhury (2000) once the transaction costs are taken into account is less

\[ \text{Recall that in this scenario, where no transaction cost is considered, the l.h.s. of relationship (6) is just the negative of the l.h.s. of equation (5) and thus also the percentages reported by Capelle-Blancard and Chaudhury (2000) sum up to one.} \]
than 4 € performing both strategy (5) and (6), while on the Italian Index Option Market a retail investor can gain on average about 185 € performing the strategy (6).

7. Conclusions
In this paper we examine the internal market efficiency of the Italian index option market, which is based on tests of arbitrage pricing relationship involving options only. Specifically we test call/put spreads, call/put butterfly spreads and box spreads involving Mibos in the period 1 September – 31 December 2002.

When we ignore transaction costs and bid-ask spreads, we report violations which display a low frequency and a disparate average amount depending on the specific strategy under analysis. These results contrast those obtained by Ackert and Tian (1998, 2001) for North American markets, whereby higher frequencies are reported, and are very much in line with the study by Capelle-Blanchard and Chaudhury (2000) on the French index option market.

In the most realistic scenario including both transaction costs and the bid-ask spread, the results we obtain for the Italian market display a common pattern. Again the frequencies are much lower than those on the US or Canadian markets and very similar to those characterising the French market. By contrast, the average amount of the violations is on the Italian market higher than on both the North American and the French market. Given the limited number of arbitrage violations a deep analysis of the determinants of the violations is not viable. However, based on an explorative analysis of the options allowing for arbitrage opportunities a possible determinant is the option maturity, whereby longer maturity options are generally associated with arbitrage opportunities with an important average amount of profits. On the other hand, the ex-ante analysis highlights that the arbitrage opportunities do not normally survive long enough to be really exploited.

In sum, the present study completes previous research-work on the cross-market efficiency of the Mibo market and points at a high level of internal efficiency. This reinforces the efficiency results obtained in previous studies.
References


Chance D. M. (1985) "Empirical Tests of the Pricing of Index Call Options." Advances in Futures and Options Research, 1, 141-166.


Figure 1
*Mibo contracts traded every year from 1995 to 2002*
Data source: Borsa Italia S.p.A

Figure 2
*Isoα and Mibo contracts per year: volumes*
Data source: Borsa Italia S.p.A

Figure 3
*Isoα and Mibo contracts per year: notional values*
Data source: Borsa Italia S.p.A
Table 1: Frequency of violations in Scenario A, by month*.

<table>
<thead>
<tr>
<th>Spread</th>
<th>September</th>
<th>October</th>
<th>November</th>
<th>December</th>
<th>Whole sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call Spread</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>(0.0%)</td>
<td>(0.14%)</td>
<td>(0.0%)</td>
<td>(0.0%)</td>
<td>(0.018%)</td>
</tr>
<tr>
<td>Put Spread</td>
<td>41</td>
<td>2</td>
<td>6</td>
<td>23</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>(0.58%)</td>
<td>(0.03%)</td>
<td>(0.08%)</td>
<td>(0.42%)</td>
<td>(0.27%)</td>
</tr>
<tr>
<td>Call Butterfly spread</td>
<td>12</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>(0.68%)</td>
<td>(0.25%)</td>
<td>(0.14%)</td>
<td>(0.21%)</td>
<td>(0.34%)</td>
</tr>
<tr>
<td>Put Butterfly spread</td>
<td>14</td>
<td>6</td>
<td>4</td>
<td>5</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>(0.84%)</td>
<td>(0.43%)</td>
<td>(0.24%)</td>
<td>(0.36%)</td>
<td>(0.47%)</td>
</tr>
<tr>
<td>Box Spread</td>
<td>32</td>
<td>19</td>
<td>11</td>
<td>23</td>
<td>85</td>
</tr>
<tr>
<td>(a)</td>
<td>(12.12%)</td>
<td>(10.11%)</td>
<td>(7.01%)</td>
<td>(8.39%)</td>
<td>(9.60%)</td>
</tr>
<tr>
<td>Box Spread</td>
<td>232</td>
<td>169</td>
<td>146</td>
<td>251</td>
<td>800</td>
</tr>
<tr>
<td>(b)</td>
<td>(87.88%)</td>
<td>(89.88%)</td>
<td>(92.99%)</td>
<td>(91.61%)</td>
<td>(90.40%)</td>
</tr>
</tbody>
</table>

* = The table reports, for each month of the period under analysis and for the whole sample, the number and the percentage (in parenthesis) of violations recorded for each no-arbitrage condition tested.

Table 2: Average amount of violations of (1)-(6) relationships, in Scenario A*.

<table>
<thead>
<tr>
<th>Spread</th>
<th>Butterfly spread</th>
<th>Box Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call</td>
<td>Put</td>
<td>Call</td>
</tr>
<tr>
<td>1310.72</td>
<td>62.01</td>
<td>17.01</td>
</tr>
</tbody>
</table>

* = The table reports the average amount of the violations (expressed in €), for each no arbitrage condition tested.
Table 3: Frequency of violations in Scenario B, by month*.

<table>
<thead>
<tr>
<th>Spread</th>
<th>September</th>
<th>October</th>
<th>November</th>
<th>December</th>
<th>Whole sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call Spread</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>(0.0%)</td>
<td>(0.07%)</td>
<td>(0.0%)</td>
<td>(0.0%)</td>
<td>(0.018%)</td>
</tr>
<tr>
<td>Put Spread</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>(0.01%)</td>
<td>(0.0%)</td>
<td>(0.01%)</td>
<td>(0.0%)</td>
<td>(0.008%)</td>
</tr>
<tr>
<td>Call Butterfly spread</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.0%)</td>
<td>(0.00%)</td>
<td>(0.00%)</td>
<td>(0.07%)</td>
<td>(0.016%)</td>
</tr>
<tr>
<td>Put Butterfly spread</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>(0.06%)</td>
<td>(0.07%)</td>
<td>(0.00%)</td>
<td>(0.00%)</td>
<td>(0.033%)</td>
</tr>
<tr>
<td>Box Spread</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(a)</td>
<td>(0.00%)</td>
<td>(0.00%)</td>
<td>(0.00%)</td>
<td>(0.00%)</td>
</tr>
<tr>
<td>Box Spread</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>(b)</td>
<td>(0.38%)</td>
<td>(0.00%)</td>
<td>(1.27%)</td>
<td>(0.34%)</td>
</tr>
</tbody>
</table>

* = The table reports, for each month of the period under analysis and for the whole sample, the number and the percentage (in parenthesis) of violations recorded for each no-arbitrage condition tested.

Table 4: Average amount of violations in Scenario B*.

<table>
<thead>
<tr>
<th>Spread</th>
<th>Call</th>
<th>Put</th>
<th>Butterfly spread</th>
<th>Call</th>
<th>Put</th>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>777.77</td>
<td>823.72</td>
<td>0.45</td>
<td>29.69</td>
<td>0</td>
<td>974.21</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* = The table reports the average amounts of the violations (expressed in €), for each no arbitrage condition tested.
**Table 5**: Frequency of violations of Call and Put Spreads in Scenario C, by month*.

<table>
<thead>
<tr>
<th></th>
<th>Call Spreads</th>
<th></th>
<th>Put Spreads</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TC=1</td>
<td>TC=10</td>
<td>TC=25</td>
<td>TC=40</td>
</tr>
<tr>
<td>Sept.</td>
<td>0 (0.00%)</td>
<td>0 (0.00%)</td>
<td>0 (0.00%)</td>
<td>0 (0.00%)</td>
</tr>
<tr>
<td>Oct.</td>
<td>5 (0.07%)</td>
<td>5 (0.07%)</td>
<td>5 (0.07%)</td>
<td>5 (0.07%)</td>
</tr>
<tr>
<td>Nov.</td>
<td>0 (0.00%)</td>
<td>0 (0.00%)</td>
<td>0 (0.00%)</td>
<td>0 (0.00%)</td>
</tr>
<tr>
<td>Dec.</td>
<td>0 (0.00%)</td>
<td>0 (0.00%)</td>
<td>0 (0.00%)</td>
<td>0 (0.00%)</td>
</tr>
<tr>
<td>Whole sample</td>
<td>5 (0.018%)</td>
<td>5 (0.018%)</td>
<td>5 (0.018%)</td>
<td>5 (0.018%)</td>
</tr>
</tbody>
</table>

* = The table reports, for each month and for the whole sample, the number and the percentage (in parenthesis) of violations recorded testing the arbitrage pricing relationships (1) and (2).
## Table 6: Frequency of violations of Call and Put Butterfly Spreads in Scenario C, by month*.

<table>
<thead>
<tr>
<th>Call Butterfly Spreads</th>
<th>Put Butterfly Spreads</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC=1</td>
<td>TC=10</td>
</tr>
<tr>
<td>TC=25</td>
<td>TC=40</td>
</tr>
<tr>
<td>TC=1</td>
<td>TC=10</td>
</tr>
<tr>
<td>TC=25</td>
<td>TC=40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>TC=1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TC=10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TC=25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TC=40</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(0.00%)</th>
<th>(0.00%)</th>
<th>(0.00%)</th>
<th>(0.00%)</th>
<th>(0.06%)</th>
<th>(0.00%)</th>
<th>(0.00%)</th>
<th>(0.00%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* = The table reports, for each month and for the whole sample, the number and the percentage (in parenthesis) of violations recorded testing the arbitrage pricing relationship (3) and (4).

## Table 7: Frequency of violations of Box Spreads in Scenario C, by month*.

<table>
<thead>
<tr>
<th></th>
<th>TC=1</th>
<th>TC=10</th>
<th>TC=25</th>
<th>TC=40</th>
<th>TC=1</th>
<th>TC=10</th>
<th>TC=25</th>
<th>TC=40</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>TC=1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TC=10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TC=25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TC=40</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(0.00%)</th>
<th>(0.00%)</th>
<th>(0.00%)</th>
<th>(0.00%)</th>
<th>(0.38%)</th>
<th>(0.38%)</th>
<th>(0.38%)</th>
<th>(0.38%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* = The table reports, for each month and for the whole sample, the number and the percentage (in parenthesis) of violations recorded testing the arbitrage pricing relationship (5) and (6).
**Table 8:** Average amount of violations of (1)-(6) relationships, in Scenario C*.

<table>
<thead>
<tr>
<th>Transaction costs</th>
<th>Spread</th>
<th>Butterfly spread</th>
<th>Box Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Call</td>
<td>Put</td>
<td>Call</td>
</tr>
<tr>
<td>TC=1</td>
<td>775.77</td>
<td>628.43</td>
<td>0</td>
</tr>
<tr>
<td>TC=10</td>
<td>757.77</td>
<td>626.43</td>
<td>0</td>
</tr>
<tr>
<td>TC=25</td>
<td>727.77</td>
<td>578.43</td>
<td>0</td>
</tr>
<tr>
<td>TC=40</td>
<td>697.77</td>
<td>548.43</td>
<td>0</td>
</tr>
</tbody>
</table>

* = The table reports the average amount of the violations (expressed in €), for each no arbitrage condition tested.