



Università degli Studi di Modena e Reggio Emilia  
Dipartimento di Economia Politica



## Materiali di discussione

\\ 518 \\

### **Economic Growth Rates and Recession Probabilities: the predictive power of the term spread in Italy**

by

Marianna Brunetti<sup>1</sup>  
Costanza Torricelli<sup>2</sup>

December 2005

<sup>1</sup> Università degli Studi di Bergamo  
Dipartimento di Matematica, Statistica, Informatica e Applicazioni  
Via Caniana,2  
24127 Bergamo (Italia)  
e-mail: [marianna.brunetti@unibg.it](mailto:marianna.brunetti@unibg.it)

<sup>2</sup> Università di Modena e Reggio Emilia  
Dipartimento di Economia Politica  
Via Berengario,51  
41100 Modena, Italy  
e-mail: [costanza.torricelli@unimore.it](mailto:costanza.torricelli@unimore.it)



**Economic Growth Rates and Recession Probabilities:  
the predictive power of the term spread in Italy.**

*Marianna Brunetti, University of Bergamo*

*Costanza Torricelli, University of Modena and Reggio Emilia\* §*

**Abstract**

The aim of the present work is to test the predictive power of the term spread in forecasting real economic growth rates and recession probabilities in Italy. According to the most recent literature, the relationship between the term spread and economic growth rates is modelled as a nonlinear one and specifically the Logistic Smooth Transition model is used, while a probit model is implemented to forecast recession probabilities. In both applications evidence supports a relevant informative content of the spread in Italy.

**Keywords :** term structure, term spread, regime prediction

**JEL Classification:** E32, E43, C53

---

\* Corresponding author: Costanza Torricelli, Dipartimento di Economia Politica, Viale Berengario 51, 41100 Modena, Italy, Tel. +39 059 2056733 Fax +39 059 2056937, e-mail:[torricelli.costanza@unimore.it](mailto:torricelli.costanza@unimore.it)

§The authors gratefully acknowledge support from MIUR and wish to thank Michael P. Clements for helpful suggestions on a previous version of the present work. Usual caveats apply.

## 1. – Introduction

Information about the future economic performance of a country is of uttermost importance in a number of applications. Policy makers need forecasts on future economic growth rates in order to design the correct stance of their policies. In finance, a field of application can be represented by the international accord known as Basel II, which sets, within a broader regulatory framework, new and more risk-sensitive capital requirements that naturally depend on the state of the economy<sup>1</sup>.

The Term Structure of Interest Rates (TSIR) and in particular the term spread, i.e. difference between long- and short-term interest rates, is a largely accepted indicator of market expectations about future economic performances. It is particularly attractive for this purpose as TSIR data are instantaneously available also for long maturities, so that forecasts are possible over long horizons as well.

The predictive power of the term spread about future economic performances basically stems from the Rational Expectation Hypothesis (EH), according to which long-term interest rates are averages of appropriate expected future short-term interest rates. In particular, when the market foresees a recession, a reduction in expected future short-term interest rates is anticipated and the TSIR flattens, so that a change in the slope of TSIR (i.e. in the term spread) indicates a change in the expected future economic performances.

The EH connection between the term spread and future real activity may be affected through two main channels: monetary policy and intertemporal consumers choices. Consider a tightening monetary policy: short-term interest rates rise, whereas long-term rates also rise but generally less than the former, leading to a reduction of the term spread. The contraction can induce lower spending in sensitive sectors of the economy and thus a slowdown in the economic growth rates (see Estrella (2005) for a comprehensive

---

<sup>1</sup> For example, in Pederzoli and Torricelli (2005) a regime prediction is used to estimate default probabilities and hence capital requirements within the Basel II framework.

theoretical rational expectations model and Estrella and Mishkin (1997) for empirical evidence in favour of the key role played by the monetary policy in the relationship between the TSIR and future real output). On the other hand, intertemporal consumer choice theory assumes that consumers prefer stable rather than fluctuating levels of income. Accordingly, if a recession is expected consumers will increase savings and buy long-term bonds to get payoffs during the slowdown, inducing a decrease of long-term yields. On the other hand, they may sell short-term bonds making the relative yields rise. Therefore, when a recession is expected, the term spread reduces and the TSIR flattens (see Harvey (1988) for a full account).

Many empirical works in literature deal with the spread as a predictor of future economic evolution but only a few have analysed this issue for the Italian case: e.g. Estrella and Mishkin (1997) and Artis et al. (2004) perform comparative studies, Moneta (2003) checks for consistency between Euro area and individual countries, Marotta et al. (2005) forecast recession likelihood to estimate default probabilities.

In order to contribute to the literature, the present work aims to test the predictive power of the spread in Italy. The present analysis differs from previous works on the Italian case for the following feature. First, two approaches are implemented in order to test the robustness of the informative content of the term spread. In the former, the term spread is used as explanatory variable of future growth rates of real economy and specifically a nonlinear model is implemented, namely the Logistic Smooth Transition (LSTR) model. In the latter the spread is used to predict the likelihood of future recessions and a binary probit model is employed for the prediction of recession probabilities. Second, a more recent and a higher-frequency dataset is used. More precisely, monthly rather than quarterly data are used, so that a closer match between the business cycle chronology and the classification of recession/expansion periods in the sample under analysis is possible. Finally, a different

business cycle chronology is adopted, i.e. the OECD one, in order to assess the sensitivity of the results to the chronology used.

The remainder of the paper is organized as follows. Section 2 briefly reviews the literature on the predictive power of the term spread over economic growth rates and regime probabilities. Section 3 presents the econometric framework used to test the predictive power of the spread. Section 4 describes the dataset, the empirical analyses and discusses the results obtained. Section 5 reports probit in- and out-of-sample forecast evaluations and compares results with literature. Section 6 concludes.

## **2. – Literature Overview**

Numerous studies provide evidence on the predictive content of the term spread for real output<sup>2</sup>. In particular, earlier works test the predictive power of the spread w.r.t. economic growth rates by means of simple linear models. Among others, Harvey (1989) reports that US real GNP growth rates 1- to 5-quarter ahead significantly depend on the contemporaneous values of the spread between 5-year TBond and 3-month TBill rates. Similarly, Estrella and Hardouvelis (1991) using US quarterly data observe that the slope of the TSIR measured by the spread between 10-year TBond and 3-month T-Bill rates predicts quite well both cumulative changes in real GNP and recession probabilities up to 4 years ahead. Cozier and Tkacz (1994) conclude that the spread predictive power on the changes in Canada real GDP is robust to the inclusion of additional informative variables (e.g. M1, real stock prices, Canada Leading Indicator and the output-gap). However, empirical evidence on the informative power of the spread is not always consistent between countries: Plosser and Rouwenhorst (1994) for instance confirm the predictive power of the spread for US, Canada and Germany, but not for France and UK.

---

<sup>2</sup> See Stock and Watson (2003) among others for an extensive survey of the literature.

By contrast, more recent works on this issue implement nonlinear models. Among others, Galbraith and Tkacz (2000) use quarterly data for the G7 countries and report empirical evidence of an asymmetric impact on the conditional expectations of output growth rates for US and Canada. They conclude that nonlinear smooth transition (STR) models with different regimes can be valuable to model this relationship and can help understand the impact of a regime shift on the relationship between output changes and the spread<sup>3</sup>. Similarly, Venetis et al. (2003) employ a Smooth Transition model and find evidence of a strong threshold effect: the relationship between the spread and economic growth rates is stronger if past spread values do not exceed a given positive value. Finally, based on a rational expectation model, Estrella (2005) proves both theoretically and empirically that the relationship between changes in real output and the term spread depends on the coefficients of the monetary reaction function. In particular, the more adverse the policy maker to deviations from target inflation, the weaker the predictive power of the spread on future output changes. In other words, this relationship is not linear as it depends, at least partially, on the monetary regime in use.

As for the predictive power of the spread over future recessions, Estrella and Mishkin (1997) study the issue for France, Germany, Italy, UK and US and find different evidence depending on the country considered: stronger predictive power in US and Germany, weaker in UK and Italy. Dueker (1997) concludes that the spread not only can provide useful information about the likelihood of future US recessions, but it also outperforms other variables, although it can predict neither the precise onset nor the duration of the recessions. Similarly, Bernard and Gerlach (1998) find evidence of the spread predictive power on future recession probabilities up to two years ahead in eight countries (Belgium, Canada, France, Germany, Japan, the Netherlands, UK and US over the period 1972-1993).

---

<sup>3</sup> Bec et al. (2002) find that the empirical description of monetary policy by linear Taylor rules sensibly improves using a STR form.

They also test its robustness to the inclusion of countries' leading indicators and report a "cross-country" effect: German and US spreads are particularly significant also in Japan and UK regressions respectively. Sédillot (2001) compares the "quantitative approach" that uses the spread to forecast economic growth rates with the "qualitative" one, in which the spread is instead used to forecast recession probabilities, and concludes that for all countries considered (France, Germany and US) the latter provides an interesting alternative to the previous one. Moneta (2003) finds evidence in favour of the spread predictive power on future recession probabilities in the whole Euro area. Finally, in Marotta et al. (2005), recession probabilities are estimated employing a probit model with both domestic and international financial variables. They find that forecasts based on the ISAE (Istituto di Studi e Analisi Economica) chronology are improved if the ECRI (Economic Cycle Research Institute) chronology is adopted and underline the importance of a further analysis of the chronology selection.

### 3. – The methodology

#### 3.1 – The spread as predictor of economic growth rates

Provided that Expectation Hypothesis holds<sup>4</sup>, the predictive power of the term spread w.r.t. future evolution of real economy can be tested by means of the following linear model:

$$\Delta y_t^k = \mathbf{a}_0 + \sum_i \mathbf{b}_i s_{t-i} + u_t \quad (1)$$

where  $y_t$  is the log of a measures of the economy performance at time  $t$  and  $\Delta y_t^k$  is the annualized growth rate of the economy over the next  $k$  periods,  $s_{t-i}$  is the  $i$ -th lag of the spread between long- and short-term interest rates and  $u_t$  is the disturbance term.

---

<sup>4</sup> EH can be tested in different ways ranging from simple regressions to cointegration tests (e.g. see Campbell and Shiller (1991), Boero and Torricelli (2002) and Sarno et al. (2005)). Here, a Johansen's procedure has been implemented on interest rates prior to all other analyses. Evidence of cointegration and thus of EH to hold in Italy was find. Detailed results for this analysis are available upon request.

However, model (1) is too simple to fully capture the nature of relationship between the spread and economic growth rates<sup>5</sup>, which is in fact characterized by nonlinearities either in form of asymmetries (i.e. the relationship differs depending on past values of the spread being positive or negative) and/or of regime switching behaviour (i.e. the informational content of the spread changes with the regime in operation). In order to capture these potential nonlinearities, the Smooth Transition (STR) model can be suitably employed:

$$\Delta y_t^k = \mathbf{a} + \sum_i \mathbf{b}_i s_{t-i} + \left( \mathbf{d} + \sum_i \mathbf{f}_i s_{t-i} \right) G(\mathbf{g}, s_{t-d}, c) + u_t \quad (2)$$

where  $G(\mathbf{g}, s_{t-d}, c)$  is the transition function which incorporates the nonlinearity of the model.  $G$  is bounded between 0 and 1 and its value depends on three different factors: (i) the slope or smoothness parameter  $\mathbf{g} > 0$ , that measures the speed of transition from one regime to another; (ii) the transition variable  $s_{t-d}$ , represented here by the spread<sup>6</sup>, whose value  $d$  periods back determines the current operating regime, and (iii) the threshold  $c$ , which in a two-regime STR model is a value such that if  $s_{t-d}$  lies below  $c$  the first regime operates, otherwise the second or alternative regime is activated.  $G$  can be either a logistic function:

$$G(\mathbf{g}, s_{t-d}, c) = \left( \frac{1}{1 + \exp\left\{ \frac{-\mathbf{g}(s_{t-d} - c)}{s_{t-d}} \right\}} \right) \quad (3)$$

or an exponential function:

---

<sup>5</sup> See among others Galbraith and Tkacz (2000) and Venetis et al. (2003).

<sup>6</sup> Along with the spread, Venetis et al. (2003) consider several other variables as potential transition variables, such as past growth rates in aggregate economic activity, quarterly output-gap and time. However, as the null of linearity is rejected using all the variables and “*the strongest rejections correspond to the spread [...]*”, they “*finally retain the lagged spread as the transition variable*”.



$$G(\mathbf{g}, s_{t-d}, c) = \left( 1 - \exp \left\{ - \frac{\mathbf{g}(s_{t-d} - c)^2}{\mathbf{s}_{s_{t-d}}^2} \right\} \right) \quad (4)$$

where in both cases  $\mathbf{s}_{s_{t-d}}$  represents the standard error of the transition variable. Thus, depending on the specification of  $G$ , model (2) can either be a Logistic Smooth Transition (LSTR) or an Exponential Smooth Transition (ESTR) model. The LSTR asymmetry depends on the threshold  $c$ , which can be 0 or any other positive or negative value. Similarly, ESTR is symmetric w.r.t  $c$  because it displays the same dynamics for values of  $s_{t-d}$  far higher and lower than  $c$  and a different one for values of  $s_{t-d}$  nearby  $c$ . The choice between LSTR and ESTR can be theoretically and/or empirically grounded. Theoretically the former seems more suitable for modelling the relationship under analysis because high spreads typically suggest increasing economic growth while low spreads usually point at a growth slowdown. Nevertheless, as in Venetis et al. (2003), the choice can be made empirically by testing the following sequence of null hypotheses:

$$H_0^1 : \mathbf{b}_{3i} = 0 \quad (5)$$

$$H_0^2 : \mathbf{b}_{2i} = 0 \mid \mathbf{b}_{3i} = 0 \quad (6)$$

$$H_0^3 : \mathbf{b}_{1i} = 0 \mid \mathbf{b}_{2i} = \mathbf{b}_{3i} = 0 \quad (7)$$

on the auxiliary regression:

$$\Delta y_t^k = \mathbf{b}_{00} + \sum_i \left( \mathbf{b}_{0i} s_{t-d} + \mathbf{b}_{1i} s_{t-i} s_{t-d} + \mathbf{b}_{2i} s_{t-i} s_{t-d}^2 + \mathbf{b}_{3i} s_{t-i} s_{t-d}^3 \right) + \mathbf{e}_t \quad (8)$$

If the p-value for the F-Statistics of  $H_0^2$  is lower than that for  $H_0^1$  and  $H_0^3$  then the exponential function is chosen, otherwise the logistic specification of  $G$  is preferred.

### 3.2 – *The spread as predictor of recession probabilities*

A second approach to test the information content of the TSIR is based on the predictability view of the business cycle and uses the term spread to predict economic recession  $k$  periods ahead.

The dependent variable used in this case, named *recession*, is an indicator variable assuming value 1 if the economy is in a recession and 0 otherwise. Following Estrella and Hardouvelis (1991) and Estrella and Mishkin (1997), a probit model can be used<sup>7</sup>:

$$P(\text{recession}_t) = F(\mathbf{a}_0 + \mathbf{a}_1 s_{t-k}) \quad (9)$$

where  $F$  indicates the normal cumulative distribution function. If  $\mathbf{a}_1$  is statistically significant, then the spread contributes to predict future recessions' probabilities and fitted values are the estimated probabilities of the economy being in a recession  $k$  periods ahead conditional on the information in the current term spread.

In order to test the robustness of the predictive power of the spread, the role of additional variables can be tested by means of the following regression:

$$P(\text{recession}_t) = F(\mathbf{a}_0 + \mathbf{a}_1 s_{t-k} + \mathbf{a}_2 X_{t-k}) \quad (10)$$

where  $X_{t-k}$  is an - or a series of - additional explanatory variable. If  $\mathbf{a}_1$  is significant in (9) but not in (10), then the predictive power of the spread is not robust to the inclusion of other informative variables.

Finally, the contribution of the spread in predicting future recessions' probabilities is evaluated on the basis of in- and out-of-sample forecasts. To this end, forecast performances of model (10) are compared with those of a benchmark model including the LI only, i.e.:

---

<sup>7</sup> A logit model could alternatively be used (as in Artis et al. (2004)). In this paper a logit model was estimated on the same dataset with similar results.

$$P(\text{recession}_t) = F(\mathbf{a}_1 + \mathbf{a}_2 LI_{t-k}) \quad (11)$$

The in-sample forecasts of models (10) and (11) are compared on the basis of the number of Hits (i.e. the model predicts recession when there is indeed recession) and of False Alarms (i.e. the model predicts recession when it does not occur). The out-of-sample forecast performances of the two models are compared by means of three measures: the Quadratic Probability Score (QPS), the Log Probability Score (LPS) and the Kuipers Score (KS). QPS is a loss function bounded between 0 and 2 defined as:

$$QPS = \frac{2}{T} \sum_{t=1}^T (\tilde{p}_t - \text{recession}_t)^2 \quad (12)$$

LPS is a non-negative function, which penalizes large mistakes more than QPS, which is computed as follows:

$$LPS = -\frac{1}{T} \sum_{t=1}^T [(\text{recession}_t) * \ln(\tilde{p}_t) + (1 - \text{recession}_t) * \ln(1 - \tilde{p}_t)] \quad (13)$$

Finally, KS by construction penalizes “one-prediction” models, i.e. those forecasting always recession or expansion, as it is defined as the difference between the percentage of Hits (H) and the percentage of False Alarms (F), respectively computed as:

$$H = \frac{\sum_{t=1}^T (\text{recession}_t) * I(\tilde{p}_t \geq \bar{p})}{\sum_{t=1}^T (\text{recession}_t)} \quad \text{and} \quad F = \frac{\sum_{t=1}^T (1 - \text{recession}_t) * I(\tilde{p}_t \geq \bar{p})}{\sum_{t=1}^T (1 - \text{recession}_t)} \quad (14)$$

where  $\bar{p}$  is a threshold value (bigger than the sample proportion) such that for  $\tilde{p} \geq \bar{p}$  the model predicts recession.

#### 4. - Dataset and Empirical Results

The dataset<sup>8</sup> spans over the period December 1983 - July 2005 and includes monthly observations for four variables in Italy: the spread, the OECD Composite Leading Indicator, a proxy for the economic activity and a dummy variable for the recession. A few observations are here in order. First, different measures of the term spread have been proposed in literature (e.g. see Harvey (1989) and Dueker (1997)). This paper sticks to the most widespread one: the spread between 10-year and 3-month rates, whereby the former is represented by the 10-year Italian Government Bond Yield and the latter by the 3-month Eurorate. Second, as a proxy for real activity the seasonally adjusted Index of Industrial Production has been preferred to the GDP since data for the latter are available only on a quarterly basis. Finally, the dummy variable for recession has been created according to the OECD chronology (see Table 1), assigning to each month in the sample value 1 if falling within a recession, i.e. between a peak and a trough, and 0 otherwise<sup>9</sup>.

**Table 1:** Turning points of Italian business cycle\*.

| Italy          |               |                         |
|----------------|---------------|-------------------------|
| Peak           | Trough        | Duration<br>(in months) |
| --             | May 1983      | --                      |
| August 1984    | January 1987  | 30                      |
| December 1989  | April 1991    | 16                      |
| September 1991 | December 1993 | 27                      |
| December 1995  | May 1999      | 41                      |
| December 2000  | November 2001 | 11                      |
| July 2002      | May 2003      | 11                      |
| January 2004   | --            | --                      |

\*=source OECD (see [www.oecd.org](http://www.oecd.org))

<sup>8</sup> Data source: Datastream.

<sup>9</sup> Since a precise dating of recessions is quite difficult, different sources usually provide different chronologies: see for a comparison the chronologies proposed by Euro Area Business Cycle Network (EABCN), Economic Cycle Research Institute (ECRI) and Istituto di Studi e Analisi Economica (ISAE). As stressed in Moneta (2003) and in Marotta et al. (2005), results are sensible to the chronology considered.

#### 4.1 - The spread as predictor of economic growth rates

As a first step, the basic linear model (1) was estimated over four forecast horizons ( $k=3,6,12,24$  months) and including six lags of the term spread ( $i=1,3,6,12,18,24$  months). Note that, given the monthly frequency of the data, the annualized rate of growth over next  $k$  periods is  $\Delta y_t^k = (1200/k) * (y_{t+k} - y_t)$ . Overall OLS estimates for  $\mathbf{b}_i$  are neither correctly signed nor statistically significant and regression  $R^2$  turn out to be very low for each specification of model (1). Furthermore, two nonlinearity tests, namely the RESET and the Luukkonen, Saikkonen and Teräsvirta (1988) test (see Appendix), reject the null of linearity at least at a 5% level of significance<sup>10</sup>. Thus, in line with the most recent literature, the following nonlinear model is implemented:

$$\Delta y_t^k = \mathbf{a} + \sum_i \mathbf{b}_i s_{t-i} + \left( \mathbf{d} + \sum_i \mathbf{f}_i s_{t-i} \right) G(\mathbf{g}, s_{t-d}, c) + u_t \quad (2)$$

where  $\mathbf{g}$  is the speed of adjustment between one regime and the other,  $d$  is the delay parameter<sup>11</sup>,  $c$  the threshold and  $\mathbf{s}_{s_{t-d}}$  the standard error of the delayed spread. Model (2) could either be a LSTR or an ESTR depending on the transition function  $G$  being respectively logistic or exponential. Even if the former seems theoretically more appropriate, as in Venetis et al. (2003) the final choice is carried out empirically by testing the sequence of null hypotheses (5)-(7) on the auxiliary regression (8). Consistently with what suggested by theory, the logistic specification for  $G$  is chosen as the p-values for  $H_0^2$  F-test are systematically bigger than those for the other two hypotheses (see Table 2).

---

<sup>10</sup> Detailed results for these analyses are available upon request.

<sup>11</sup> For the determination of the delay parameter  $d$  see the Appendix.

**Table 2:** F-statistics and p-values for the choice of the transition function.

| k  | d  | $H_0^1 : \mathbf{b}_{3i} = 0$ |         | $H_0^2 : \mathbf{b}_{2i} = 0 \mid \mathbf{b}_{3i} = 0$ |         | $H_0^3 : \mathbf{b}_{1i} = 0 \mid \mathbf{b}_{2i} = \mathbf{b}_{3i} = 0$ |         |
|----|----|-------------------------------|---------|--|---------|--|---------|
|    |    | F-stat                        | p-value | F-stat   | p-value | F-stat   | p-value |
| 3  | 3  | 3.057                         | 0.007   | 1.620  | 0.143   | 1.735  | 0.114   |
| 6  | 2  | 5.497                         | 0.000   | 5.342  | 0.000   | 6.661  | 0.000   |
| 12 | 1  | 10.212                        | 0.000   | 2.804  | 0.012   | 10.611   | 0.000   |
| 24 | 11 | 9.624                         | 0.000   | 2.371  | 0.031   | 3.307  | 0.004   |

Thus, the nonlinear model estimated with Nonlinear Least Squares (NLS) is specified as:

$$\Delta y_t^k = \mathbf{a} + \sum_i \mathbf{b}_i s_{t-i} + \left( \mathbf{d} + \sum_i \mathbf{f}_i s_{t-i} \right) \left( \frac{1}{1 + \exp \left\{ \frac{-\mathbf{g}(s_{t-d} - c)}{\mathbf{s}_{s_{t-d}}} \right\}} \right) + u_t \quad (2')$$

Estimates are expected to be: positive for  $\mathbf{b}_i$  and negative for  $\mathbf{f}_i$ . In other words, if the lagged value of the spread is lower than  $c$ , i.e. the first regime is activated, an increase in the spread points to an increase in the economic activity, while if the second regime is active (i.e. if the spread is already exceptionally high and above the positive threshold  $c$ ) an additional increase in the spread leads to a reduction in economic growth.

**Table 3:** Initial estimates for  $\mathbf{g}$ .

| Forecast horizon | Estimates |
|------------------|-----------|
| k=3              | 258.0350  |
| k=6              | 282.5378  |
| k=12             | 289.2488  |
| k=24             | 270.3917  |

A general-to-specific approach is adopted to select the significant spreads: all lagged spreads ( $i=1,3,6,12,18,24$  months) are initially included, then the non-significant ones are sequentially eliminated and the nonlinear models re-estimated till the appropriate final specifications are found. As the initial NLS estimates for  $\mathbf{g}$  (see Table 3) are always very

high, indicating that only a few observations are actually near the threshold  $c$ , they are replaced with a ceiling value of 100 and the models are re-estimated<sup>12</sup>.

Table 4 reports NLS estimation outputs of the final specifications for LSTR model (2') in their for each forecast horizon  $k=3,6,12,24$  months. Regardless of  $k$ , the most significant coefficients are associated with the last 6-month, one-year and two-year spreads and all significant coefficients have quite high magnitudes. Italian data thus validate what suggested by economic theory: i.e. the term spread has a significant role as an explanatory variable of economic growth rates, even if with some delay. Furthermore, most  $b_i$  coefficients have negative sign (with the only exception of last-year spread when  $k=12$ ) and  $f_i$  positive. This seems reasonable given the negative estimates for the threshold  $c$  that increases with the forecast horizon from -2.21 to a value not significantly different from 0. These results thus prove the existence of a threshold relationship between the spread and expected economic growth rates: in other words, evidence in favour of the informative content of the term spread is provided and the need for a nonlinear specification is validated. In fact, the ratio  $s_{NL}/s_L$  is always less than one and thus indicates the nonlinear specification as more appropriate than the linear one.

---

<sup>12</sup> The procedure is in line with Venetis et al. (2003). It could lead to inconsistent estimates; however, the bias is practically negligible provided that  $g$  is sufficiently large.

**Table 4:** Estimation output of the LSTR model<sup>§</sup>.

$$Model \quad \Delta y_t^k = \mathbf{a} + \sum_i \mathbf{b}_i s_{t-i} + \left( \mathbf{d} + \sum_i \mathbf{f}_i s_{t-i} \right) \left\{ \frac{1}{1 + \exp \left\{ \frac{-\mathbf{g}(s_{t-d} - c)}{\mathbf{s}_{s_{t-d}}} \right\}} \right\} + u_t$$

| <i>k</i>                                       | 3                | 6                      | 12               | 24                     |                  |                        |                  |              |
|--|------------------|------------------------|------------------|------------------------|------------------|------------------------|------------------|--------------|
| <i>d</i>                                       | 3                | 2                      | 1                | 11                     |                  |                        |                  |              |
| <b>Coeff.</b>                                  | <b>Estimates</b> | <b>Coeff.</b>          | <b>Estimates</b> | <b>Coeff.</b>          | <b>Estimates</b> | <b>Coeff.</b>          | <b>Estimates</b> |              |
| <b>a</b>                                       | 5.9480**         | <b>a</b>               | -0.2319          | <b>a</b>               | 2.3741***        | <b>a</b>               | 1.7122***        |              |
| <b>b</b> <sub>24</sub>                         | -2.1994***       | <b>b</b> <sub>12</sub> | -1.0965*         | <b>b</b> <sub>12</sub> | 0.5913*          | <b>b</b> <sub>24</sub> | -0.8469***       |              |
| <b>d</b>                                       | -4.8050*         | <b>b</b> <sub>24</sub> | -3.6323***       | <b>b</b> <sub>24</sub> | -1.2801***       | <b>d</b>               | -0.5788          |              |
| <b>f</b> <sub>6</sub>                          | 0.9434**         | <b>d</b>               | 1.5990*          | <b>d</b>               | -1.9253**        | <b>f</b> <sub>24</sub> | 0.5647***        |              |
| c  | -2.2110***       | <b>f</b> <sub>12</sub> | 1.8253***        | <b>f</b> <sub>6</sub>  | 0.8459**         | c                      | -2.4345          |              |
| -  | -                | <b>f</b> <sub>24</sub> | 1.7001***        | <b>f</b> <sub>24</sub> | 0.5493**         | -                      | -                |              |
| -  | -                | c                      | -0.3585***       | c                      | -0.1407***       | -                      | -                |              |
| R <sup>2</sup>                                 | 0.118882         |                        | 0.313220         |                        | 0.307339         |                        | 0.195372         |              |
| <b>s</b> <sub>NL</sub> / <b>s</b> <sub>L</sub> | 0.99049723       |                        | 0.96577094       |                        | 0.96278907       |                        | 0.95910198       |              |
| <b>Tests #</b>                                 | <b>Stat.</b>     | <b>Prob.</b>           | <b>Stat.</b>     | <b>Prob.</b>           | <b>Stat.</b>     | <b>Prob.</b>           | <b>Stat.</b>     | <b>Prob.</b> |
| <b>Heterosk.</b>                               | 0.9827           | 0.4550                 | 1.5101           | 0.1453                 | 1.6685           | 0.0403                 | 2.3877           | 0.0040       |
| <b>Autocorr.</b>                               | 10.361           | 0.0000                 | 12.723           | 0.0000                 | 21.796           | 0.0000                 | 48.624           | 0.0000       |
| <b>Normality</b>                               | 2.2505           | 0.3245                 | 4.5813           | 0.1011                 | 3.8206           | 0.1480                 | 3.6100           | 0.1644       |

§=\*, \*\* and \*\*\* denote a 10%, 5% and 1% level of significance respectively.

# = Just F-Stat are reported for Heteroskedasticity and Autocorrelation tests, as Obs\*R<sup>2</sup> always give similar results.

#### 4.2 - The spread as predictor of recessions' probabilities

Table 5 reports the estimation output of the probit model over forecast horizons of 3, 6, 12 and 24 months:

$$P(\text{recession}_t) = F(\mathbf{a}_0 + \mathbf{a}_1 s_{t-k}) \quad (9)$$

The coefficients associated with the spread all have the correct theoretical sign (i.e. negative) and, except for k=12, they all are strongly significant, with estimated values varying between -0.21 and -0.42. Italian data thus corroborate the existence of a significant link between the spread and recession probabilities.



**Table 5:** Estimates of Probit model (9).

|                                | <b>k=3</b> | <b>k=6</b> | <b>k=12</b> | <b>k=24</b> |
|--------------------------------|------------|------------|-------------|-------------|
| <b>a<sub>0</sub></b>           | 0.3193***  | 0.3228***  | 0.2379***   | 0.0747      |
| <b>a<sub>1</sub></b>           | -0.2394*** | -0.2124*** | -0.0335     | -0.4172***  |
| <b>RRS</b>                     | 58.50485   | 58.25507   | 59.46097    | 49.99486    |
| <b>S.E. of regr.</b>           | 0.480879   | 0.482722   | 0.493652    | 0.464214    |
| <b>Log-lik.</b>                | -165.9178  | -164.7849  | -166.4095   | -143.5784   |
| <b>Restricted Log-lik</b>      | -172.9822  | -171.9949  | -170.4948   | -143.9348   |
| <b>*McFadden R<sup>2</sup></b> | 0.041545   | 0.033242   | 0.000892    | 0.102644    |
| <b>*f</b>                      | 0.0550     | 0.0568     | 0.0331      | 0.0030      |

\* = Measures of fit typically used for binary regressions. McFadden R<sup>2</sup> is computed as  $1 - \left( \frac{\log(L_u)}{\log(L_c)} \right)$ , where  $\log(L_u)$  and  $\log(L_c)$  are respectively the unconstrained and constrained log-likelihood of the model, the latter being obtained when all the slope coefficients are constrained to zero. **f** is instead defined as  $f = 1 - \left( \frac{\log(L_u)}{\log(L_c)} \right)^{-\frac{2}{\#OBS} \log(L_c)}$ .

In order to test the robustness of this result, model (9) is re-estimated including an additional explanatory variable. While some authors (e.g Estrella and Mishkin (1997)) include into the model more than one variable, in this paper only the OECD Composite Leading Indicator (hereafter LI) is included in (10) as it already encloses several economic indicators<sup>13</sup>. Table 6 thus reports the estimation output of the following model:

$$P(\text{recession}) = F(\mathbf{a}_0 + \mathbf{a}_1 s_{t-k} + \mathbf{a}_2 LI_{t-k}) \quad (10')$$

**Table 6:** Estimates of Probit model (10').

|                                | <b>k=3</b> | <b>k=6</b> | <b>k=12</b> | <b>k=24</b> |
|--------------------------------|------------|------------|-------------|-------------|
| <b>a<sub>0</sub></b>           | -1.0956    | -0.7210    | 0.6297      | 1.3987      |
| <b>a<sub>1</sub></b>           | -0.3130*** | -0.2652*** | -0.0154     | -0.4842***  |
| <b>a<sub>2</sub></b>           | 0.0151     | 0.0111     | -0.0042     | -0.0142     |
| <b>RRS</b>                     | 58.21912   | 58.10119   | 59.43850    | 49.75950    |
| <b>S.E. of regr.</b>           | 0.480654   | 0.483051   | 0.494573    | 0.464122    |
| <b>Log-lik.</b>                | -165.1964  | -164.3997  | -166.3543   | -142.9602   |
| <b>Restricted Log-lik</b>      | -172.9822  | -171.9949  | -170.4948   | -143.9348   |
| <b>*McFadden R<sup>2</sup></b> | 0.045713   | 0.035502   | 0.001223    | 0.106507    |
| <b>*f</b>                      | 0.0606     | 0.0598     | 0.0335      | 0.0083      |

\* = See Table 5 for definitions.

<sup>13</sup> See [www.oecd.org](http://www.oecd.org) for additional information.

The coefficients associated with LI are never statistically different from zero while, consistently with model (9) the spread coefficients remain strongly significant and negatively signed in all cases but  $k=12$ . Furthermore, the inclusion of LI into the model produces only minor changes in both fit measures McFadden  $R^2$  and  $f$ , suggesting that no relevant improvement of the model is produced when LI is included. Therefore Italian data not only confirm the link existing between the term spread and future recessions' probabilities, but also prove its robustness to the inclusion of an additional informative variable such as LI.

## **5. – Forecast evaluation: a comparison with the literature**

The predictive power of the spread can be evaluated by means of the forecast performance of the above-estimated models. However, as forecasts with nonlinear LSTR model are quite cumbersome<sup>14</sup>, in this paper the focus is on the probit model only which allows for simpler but still effective forecasts.

More precisely, in- and out-of-sample forecasts of the benchmark model (11) including the LI only are compared with those of model (10') including both LI and the term spread. In order to compute the number of Hits and False Alarms, we assume that the model predicts a recession when  $\tilde{p}_t \geq 0.55$ . For the model to predict a recession, the fitted probability must increase above the sample proportion that in this case is 0.5231<sup>15</sup>. It follows that the rule taken in other papers,  $\tilde{p}_t \geq 0.5$ , cannot be adopted here since the model would always predict recessions. Hence, in order to compensate for the prudent OECD chronology, a slightly higher but still reasonable threshold is chosen.

---

<sup>14</sup> See for instance Granger and Teräsvirta (1993) and Clements et al. (2004).

<sup>15</sup> Our sample counts for 136 periods classified as recessions out of 260 since in OECD chronology also minor cycles are taken into account.

The number and proportion of Hits and False Alarms of the in-sample forecasts for both models are reported in Table 7. In all cases the model including the spread displays a higher number of Hits and a smaller (or in one case equal) number of False Alarms. Thus, in-sample forecasts confirm that the spread actually adds useful information to predict future recessions and hence substantiate its predictive power.

**Table 7:** Number and proportion of Hits and False Alarms of in-sample forecasts.

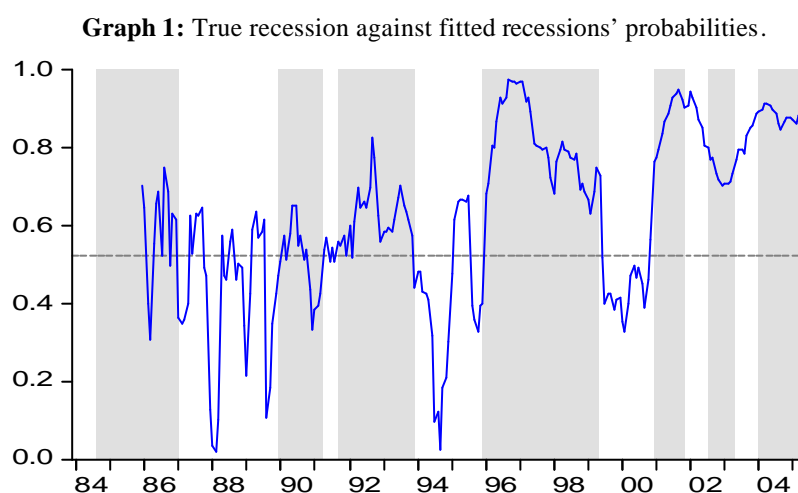
| Model        | $P(\text{recession}) = F(\mathbf{a}_0 + \mathbf{a}_2 LI_{t-k})$ |            |             |            | $P(\text{recession}) = F(\mathbf{a}_0 + \mathbf{a}_1 s_{t-k} + \mathbf{a}_2 LI_{t-k})$ |            |             |            |
|--------------|---|------------|-------------|------------|--|------------|-------------|------------|
|              | k=3   | k=6        | k=12        | k=24       | k=3  | k=6        | k=12        | k=24       |
| # Obs.       | 255   | 252        | 246         | 234        | 255  | 252        | 246         | 234        |
| Hits         | 95/149  | 103/149    | 145/145     | 91/133     | 104/149  | 109/149    | 145/145     | 93/133     |
| %            | <b>64%</b>  | <b>69%</b> | <b>100%</b> | <b>68%</b> | <b>70%</b>   | <b>73%</b> | <b>100%</b> | <b>70%</b> |
| False Alarms | 69/106  | 68/103     | 101/101     | 40/100     | 50/106   | 55/103     | 101/101     | 38/100     |
| %            | <b>65%</b>  | <b>66%</b> | <b>100%</b> | <b>40%</b> | <b>47%</b>   | <b>53%</b> | <b>100%</b> | <b>38%</b> |

Out-of-sample forecasts are computed over the period January 1995 – July 2005 and are evaluated on the basis of three measures: the Quadratic Probability Score (QPS), the Log Probability Score (LPS) and the Kuipers Score (KS). Table 8 reports a comparison between the two models. Loss-functions QPS and LPS always assume lower values in the model including the spread as well and hence the latter has additional predictive power. However, the effects of the prudential OECD chronology are confirmed by KS, which scores zero as the model always predicts recession or expansion.

**Table 8:** Measures of out-of-sample accuracy of basic and with-spread models.

| Model            | $P(\text{recession}) = F(\mathbf{a}_0 + \mathbf{a}_2 LI_{t-k})$ |        |        |         | $P(\text{recession}) = F(\mathbf{a}_0 + \mathbf{a}_1 s_{t-k} + \mathbf{a}_2 LI_{t-k})$ |        |        |         |
|------------------|---|--------|--------|---------|--|--------|--------|---------|
|                  | k=3   | k=6    | k=12   | k=24    | k=3  | k=6    | k=12   | k=24    |
| Accuracy Measure |   |        |        |         |  |        |        |         |
| QPS              | 0.9584  | 1.0016 | 0.8202 | 0.5835  | 0.9462   | 0.9800 | 0.8069 | 0.5211  |
| LPS              | 1.4392  | 1.5984 | 1.1036 | 1.0176  | 1.3878   | 1.4982 | 1.0794 | 0.7424  |
| Hits             | 0.00%   | 0.00%  | 0.00%  | 100.00% | 0.00%  | 0.00%  | 0.00%  | 100.00% |
| False Alarms     | 0.00%   | 0.00%  | 0.00%  | 100.00% | 0.00%  | 0.00%  | 0.00%  | 100.00% |
| KS               | 0.0000  | 0.0000 | 0.0000 | 0.0000  | 0.0000   | 0.0000 | 0.0000 | 0.0000  |

In sum both in- and out-of-sample forecasts prove that the term spread can provide useful information to forecast future recessions in Italy. Based on this evidence, the spread only is employed to predict future recessions and the fitted recession probabilities of model (9) are compared with actual recessions as from the OECD chronology (see Graph 1). The spread forecasts are not fully satisfactory for the period 1984-1990 but they appear more accurate starting from the beginning of 1991. The spread alone actually predicts all major recessions (91-93, 95-99, 00-01) reported also by ISAE and ECRI chronologies, gives just one False Alarm in July 1995 and captures the recurring regime of recession of last five years reported by OECD chronology.



*The shaded areas represent actual recessions as from the OECD chronology. The dotted line indicates the sample proportion (0.5231) while solid line represents the recession probabilities for each month estimated using the two-years earlier spread*

Since no previous works has empirically tested the predictive power of the spread in Italy by means of both approaches followed here, a straight comparison of our results with existing literature is not possible. However, a few recent works have tested the informative content of Italian term spread w.r.t. recession probabilities: Estrella and Mishkin (1997), Moneta (2003), Artis et al. (2004), and Marotta et al. (2005). Dataset frequency, model estimated and chronology used in each of these studies are reported in Table 9.

**Table 9:** Comparison with existing literature.

| <b>Authors</b>              | <b>Dataset<br/>period and frequency</b> |           | <b>Model</b> | <b>Chronology*</b> |
|-----------------------------|---|-----------|--------------|--------------------|
| Estrella and Mishkin (1997) | 1973 – 1994                             | Quarterly | Probit       | CCIBCR             |
| Moneta (2003)               | 1971 – 2002                             | Quarterly | Probit       | ECRI               |
| Artis et al. (2004)         | 1970 – 2001                             | Monthly   | Logit        | ECRI               |
| Marotta et al. (2005)       | 1970 – 2002                             | Quarterly | Probit       | ISAE and ECRI      |
| This study                  | 1983 – 2005                             | Monthly   | Probit       | OECD               |

\* = *CCIBCR stands for Columbia Centre for International Business Cycle Research, ECRI for Economic Cycle Research Institute and ISAE for Istituto di Studi e Analisi Economica.*

Estrella and Mishkin (1997) analyse the informative content of term spread on real activity in Italy at a comparative level with France, Germany, UK and US. As for Italian case they report that the term spread has a predictive power on recession probabilities up to one and two years ahead and the result is robust to the inclusion of other monetary indicators. Moneta (2003) tests the predictive power of the spread in Italy, France and Germany to check whether evidence for the whole Euro area, which is basically the focus of the paper, and for single countries are consistent. Even if less strong than in Germany, the author finds a significant predictive power of the term spread also in Italy and shows that the spread is more powerful than the OECD Composite Leading Indicator in forecasting recessions. Artis et al. (2004) test the predictive power of the term spread on recession probabilities three-months ahead in Italy together with Germany, France, and UK. Even if here a logistic rather than a probit model is used, a significant informative content of the term spread is reported. Marotta et al. (2005) estimate recession probabilities for an application to the Basel II capital requirement formula, performing the forecast within a probit model and comparing two different chronologies, namely ISAE and ECRI ones. In both cases evidence in favour of the term spread predictive power is found, even if forecast performance sensibly improves when ECRI chronology is adopted.

By a comparative inspection between the results in this paper and previous ones, two main remarks are in order. First, in line with the literature the predictive power of the spread is here validated, despite different approaches, dataset and chronologies are adopted. Thus,

the overall informative content of the term spread turns out to be robust to the methodology used for the empirical analysis. On the other hand some results appear to be sensitive to the setup taken in the empirical investigation (recalled in Table 9). Artis et al. (2004) observe that the predictive power of the spread is not maintained when other informative variables are considered. In contrast, the robustness of the informative content of the term spread to the inclusion of additional variables is here validated, in line Estrella and Mishkin (1997), Moneta (2003) and Marotta et al. (2005). Furthermore, our results indicate that the term spread predictive power is stronger for long forecast horizons, i.e. up to two years ahead as in Estrella and Mishkin (1997). Moneta (2003) reports instead that the informative content of the spread weakens as the forecast horizons widens.

In sum, the choices concerning the methodology, the dataset and the chronology adopted have to be taken seriously into account in interpreting results and using them for policy issues.

## **6. Conclusions and Further Research**

Many papers in the literature claim that the TSIR can provide useful information about future economic performance and that the term spread has a particular predictive power w.r.t. both growth rates and recession probabilities. Given that only a few works have analysed this issue for the Italian case, the aim of this paper is to test the predictive power of term spread in Italy.

This paper differs from the previous ones on the issue for the dataset, the business cycle chronology and the methodology used. First, a more recent and higher-frequency dataset is used, spanning over the period December 1983 – July 2005 and including monthly rather than quarterly observations, that allow a better match between the business cycle chronology and the classification of recession/expansion periods in the sample under

analysis is possible. Second, as previous works point at the sensitivity of the results to the chronology used (see Moneta (2003) and Marotta et al. (2005)), the OECD chronology, never used in previous works related to Italian case, is here adopted. Finally, two approaches are here implemented to assess the informative content of the term spread on real activity: in the first the spread is used to forecast economic growth rates and in the second it is used as predictor of future recession probabilities. As for the former the nonlinear Logistic Smooth Transition (LSTR) model is estimated implementing a general-to-specific procedure to find the best specification for each forecast horizon under analysis. As for the second approach a binary probit model is employed, using as explanatory variables either the spread alone or the spread along with the OECD Composite Leading Indicator (LI). Both approaches consistently provide evidence in favour of the term spread informative power: spread's coefficients are overall significant, especially those associated with last 1- and 2-year lag spreads, consistently with economic theory and empirical evidence generally reported in previous studies. Moreover, in- and out-of-sample probit forecast performances are evaluated, proving that the term spread can actually provide valuable information to forecast Italian business cycle and that this predictive power is robust to the inclusion of other informative variables, such as the OECD LI.

Our analyses can be extended in several ways. The LSTR model estimated, although providing a significant improvement over the linear specification, still displays some imperfections (e.g.  $s_{NL}/s_L$  ratio never below 0.95, low  $R^2$ ). Thus either a LSTR model with more than two regimes (e.g. three: high, mid and low spread values) or different and more complex nonlinear specifications can be investigated. As for the predictive power of the spread w.r.t. recession probabilities, the robustness of our results may be further tested including into the model other informative financial variables, both national (e.g. real money supply, short-term interest rates) and international (e.g. foreign spreads).

## **.References**

- Artis M., Birchenhall C., Osborn D., Sensier M. (2004), "Domestic and international influences on business cycle regimes in Europe", *International Journal of Forecasting*, 20, 343-357.
- Bec F., Ben Salem M., Collard F. (2002), "Asymmetries in Monetary Policy Reaction Function: Evidence for the U.S., French and German Central Banks", *Studies in Nonlinear Dynamics and Econometrics*, 6, 40-62.
- Bernard H., Gerlach S. (1998), "Does the Term Structure Predict Recessions? The International Evidence", *International Journal of Finance and Economics* 3, 195-215.
- Boero G., Torricelli C. (2002), "The Information in the Term Structure of German Interest Rates", *European Journal of Finance*, 8, 21-45.
- Campbell, J.Y. and R.J. Shiller (1991), "Yield Spreads and Interest Rate Movements: A Bird's Eye View", *Review of Economic Studies*, 58, 495-514.
- Clements M.P., Franses P. H. and Swanson N. R. (2004), "Forecasting Economic and Financial Time-Series with Non-Linear Models", *International Journal of Forecasting*, 20, 169-183.
- Cozier B., Tkacz G. (1994), "The Term Structure and Real Activity in Canada," *Economics Working Paper Archive at WUSTL*.
- Dueker M. J. (1997), "Strengthening the Case for the Yield Curve as Predictor of U.S. Recessions", *Federal Reserve Bank St. Louis Review*, 79, 41-51.
- Estrella A. (2005), "Why Does the Yield Curve Predict Output and Inflation?", *The Economic Journal*, 115, 722-744



- Estrella A., Hardouvelis G. (1991), 'The Term Structure as a Predictor of Real Economic Activity', *Journal of Finance*, 46, 555-576.
- Estrella A., Mishkin F.S. (1997), "The Predictive Power of the Term Structure of Interest Rates in Europe and the United States: Implications for the European Central Bank", *European Economic Review*, 41, 1375-1401.
- Galbraith J.W, Tkacz G. (2000), "Testing for Asymmetry in the Link Between the Yield Spread and Output in the G-7 Countries", *Journal of International Money and Finance*, 19, 657-672.
- Granger C.W.J., Teräsvirta T. (1993), "Modelling Nonlinear Dynamic Relationships", Oxford University Press.
- Hardouvelis G.A. (1988), "The Predictive Power of the Term Structure During Recent Monetary Regimes", *Journal of Finance*, 43, 339-56.
- Harvey C.R. (1988), "The real term structure and consumption growth", *Journal of Financial Economics*, 22, 305-334.
- Harvey C.R. (1989), "Forecasts of Economic Growth from the Bond and Stock Markets", *Financial Analysts' Journal*, 45, 38-45.
- Luukkonen R., Saikkonen P., Teräsvirta T. (1988), "Testing Linearity against Smooth Transition Autoregressive Models", *Biometrika*, 75, 491-499.
- Marotta G., Pederzoli C., Torricelli C. (2005), 'Forward-Looking Estimation of Default Probabilities with Italian Data', *Euro-Mediterranean Economics and Finance Review*, forthcoming.
- Moneta F. (2003), "Does the Yield Spread Predict Recessions in the Euro Area?", *European Central Bank Working Paper Series*, Working Paper N° 294.

- Pederzoli C., Torricelli C. (2005), "Capital requirements and Business Cycle Regimes: Forward-looking modelling of Default Probabilities", *Journal of Banking and Finance*, 29, 3121-3140.
- Plosser C.I., Rouwenhorst G.K. (1994), "International Term Structures and Real Economic Growth", *Journal of Monetary Economics*, 33, 133-155.
- Sarno L., Thornton D. L., Valente G. (2005), "The Empirical Failure of the Expectations Hypothesis of the Term Structure of Bond Yields", *Journal of Financial and Quantitative Analysis*, forthcoming.
- Sédillot F. (2001), "La pente des taux contient-elle de l'information sur l'activité économique future?", *Economie et Prévision*, 147, 141-57.
- Stock J. H. and Watson M. H. (2003), "Forecasting Output and Inflation: The Role of Asset Prices", *Journal of Economic Literature*, 41, 788-829.
- Venetis I.A., Paya I., Peel D.A. (2003), "Re-Examination of The Predictability of Economic Activity Using the Yield Spread: A Nonlinear Approach", *International Review of Economics & Finance*, 12, 187-207.

**Appendix - Luukkonen, Saikkonen and Teräsvirta test and delay parameter  $d$ .**

While the Regression Specification Error Test (RESET) tests for general types of specification errors (e.g. incorrect functional forms as well as omitted variables), the test originally proposed by Luukkonen, Saikkonen and Teräsvirta (1988), LST test hereafter, is more specifically focussed on the nonlinearity specification of the model. Note that linearity of models such as (1) could be simply verified by testing  $H_0 : \mathbf{g} = 0$  versus  $H_1 : \mathbf{g} > 0$  on (2) since under the null  $G$  is a constant and the model collapses back into its original linear specification. However, in such a case, the parameters  $c$ ,  $\mathbf{a}$ ,  $\mathbf{b}$  could assume any value, so that the model would not be identified. As a consequence, LST test needs instead to be used. According to it the linearity of models such as (1) is tested running the following auxiliary regression:

$$\Delta y_t^k = \mathbf{b}_{00} + \sum_i \left( \mathbf{b}_{0i} s_{t-d} + \mathbf{b}_{1i} s_{t-i} s_{t-d} + \mathbf{b}_{2i} s_{t-i} s_{t-d}^2 + \mathbf{b}_{3i} s_{t-i} s_{t-d}^3 \right) + \mathbf{e}_t \quad (\text{A1})$$

and then testing the following joint-significance hypothesis:

$$H_0 : \mathbf{b}_{1i} = \mathbf{b}_{2i} = \mathbf{b}_{3i} = 0 \quad (\text{A2})$$

where the delay parameter  $d$  is chosen for each horizon  $k$  as the one that minimizes the p-value (see Table 10) of the null being tested, in this case (A2). Rejection of (A2) implies non linearities, even though it does not say which kind of nonlinearity is actually missed by the model.

**Table 10:** Grid search for  $d$ , by predictive horizons  $k^*$ .

| <b>d</b> | <b>Probability</b> |               |               |        | <b>d</b> | <b>Probability</b> |        |        |               |
|----------|--------------------|---------------|---------------|--------|----------|--------------------|--------|--------|---------------|
|          | $K=3$              | $K=6$         | $K=12$        | $K=24$ |          | $K=3$              | $K=6$  | $K=12$ | $K=24$        |
| 1        | 0.0488             | 0.0053        | <b>0.0000</b> | 0.0000 | 7        | 0.0071             | 0.5646 | 0.0101 | 0.0000        |
| 2        | 0.0012             | <b>0.0000</b> | 0.0000        | 0.0000 | 8        | 0.0122             | 0.6994 | 0.0105 | 0.0000        |
| 3        | <b>0.0000</b>      | 0.0140        | 0.0000        | 0.0000 | 9        | 0.0007             | 0.9567 | 0.0681 | 0.0000        |
| 4        | 0.0000             | 0.0869        | 0.0000        | 0.0000 | 10       | 0.0043             | 0.5627 | 0.0809 | 0.0000        |
| 5        | 0.2179             | 0.1522        | 0.0003        | 0.0000 | 11       | 0.1544             | 0.6121 | 0.0994 | <b>0.0000</b> |
| 6        | 0.0211             | 0.2366        | 0.0069        | 0.0000 | 12       | 0.4465             | 0.1687 | 0.1222 | 0.0000        |

\* = bold values are the minima.