To what extent fuzzy set theory and structural equation modelling can measure functionings? An application to child well being

Tindara Addabbo,* Maria Laura Di Tommasoand Gisella Facchinetti*****

* CAPP and CHILD Department of Economics,
Faculty of Economics "Marco Biagi",University of Modena and Reggio Emilia
addabbo.tindara@unimore.it

** CHILD and Department of Economics "Cognetti de Martiis"
University of Turin
marialaura.ditommaso@unito.it

*** Department of Economics, Faculty of Economics "Marco Biagi"
University of Modena and Reggio Emilia
facchinetti.gisella@unimore.it

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Abstract

This paper explores the possibilities of using fuzzy inference system and structural equation modelling to measure capabilities both at a theoretical and empirical level.

Fuzzy set theory has been already used to measure functionings (Chiappero Martinetti 1996, 2000, Lelli 2001) while structural equation modelling has not been used till now (apart from some preliminary results on children well being in India presented by Di Tommaso 2003). In this paper we outline the major advantages and disadvantages of both the approaches both in terms of the statistical assumptions implied and in terms of their ability to measure functionings. Are the statistical assumptions implied by these approaches compatible with the capability approach? What limitations the statistical assumptions impose to the capability approach?

In order to assess to what extent these two statistical techniques work, we will apply them to measure child well being with a capabilities approach. The aim is also how to propose a list of capabilities with reference to children well being in Italy.

The applied part of the paper will use a data set based on a ISTAT (Italian National Statistical Office) multipurpose survey on family and on children condition in Italy to recover information on children's education, the socio-demographic structure of their families, child care provided by relatives and parents according to the type of activities in which the children are involved.

In the conclusion of the paper, we outline if and to what extent these statistical and fuzzy techniques can be used to measure functionings with special reference to child well being.

Introduction[1]

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Section I Capability approach and child well being

Endorsing a specific list of capabilities in the case of children well being in Western societies and in particular in Italy it is very different from endorsing a list for adults. In what follows we explain why and in which way.

Nussbaum (2003) argues that the capabilities approach should not only include the capabilities of the people who are in need (typically children or elderly) but the capability approach should endorse a theory of social justice where the

[1] We would like to thank participants to the 4th International Conference on the Capability approach (Enhancing Human Security, University of Pavia, Italy) and to the 2004 IAFFE Conference (Oxford, UK) for their stimulating comments on a previous version of this paper. The responsibility of what follows is entirely to be attributed to the authors.

subjects are not anymore only "fully cooperating members of society over a complete life"[2] .

" So I believe we need to delve deeper, redesigning the political conception of the person, bringing the rational and the animal into a more intimate relation with one another, and acknowledging that there are many types of dignity in the world, including the dignity of mentally disabled children and adults, the dignity of the senile demented elderly, and the dignity of babies at the breast.
…………………………………………………………
We thus need to adopt a political conception of the person that is more an Aristotelian than Kantian, one that sees the person from the start as both capable and needy – " in need of a rich plurality of life-activities " to use a Marx's phrase, whose availability will be the measure of well-being."[3]

In order therefore to conceptualise children well-being in Italy we consider children as subjects. In the language of rights, children rights have been established for a long time. But in the case of capabilities we are not aware of any conceptualisation.

Another theoretical problem is to assess to what extent what we observe is the result of the child's choice (given a vector of commodities and his/her capabilities) or it is determined by parents or other individuals, to what extent the functionings we observe for a given capability are just affected by parents' and social environment or are a projection of parents' ideas and of actual constraints. For instance to what extent observing that a child is attending a piano lecture is a result of his/her choice given his/her capability or it is determined by the wish of his/her parents to play piano and to have a child doing it? Probably having longitudinal data or data on the child achievements in each activities could provide us a better signal on whether the observed functioning is just a realization of parents' wishes or is an expression of child's capability (even if this is definitely connected to parents' wishes and expectations).
Moreover children's capabilities are bound to be affected not only in a static way by family, personal, social and environmental factors but the relationship with child's environment (including an extended net of relationship and of institutional constraints like for instance the availability of kindergarten or of full-time schools and the very quality of the school system) is bound to affect the very development of capabilities on the individual capability profile over his/her life cycle as the child ages. For instance if one takes the attendance of child care services its impact on children's cognitive outcomes can be observed over child's life cycle.[4]
Different environmental factors may interact in affecting child's capabilities or their conversion in functionings. Again taking the example of childcare attendance empirical analyses show that childcare attendance at early age

[2] Rawls 1980, pag 546, citation taken from Nussbaum 2003.
[3] Nussbaum 2003 pag 29-30.
[4] Walfogel (2002) provides a survey on the impact of childcare and mother's employment status on child outcomes.

produces different effects on child's cognitive ability (and therefore may differently affect child's education and knowledge capability) according to family characteristics.[5]

In this regard we believe that an interdisciplinary group with developmental psychologists, child psychoanalysts, pedagogists can improve our understanding of the links amongst capabilities over the life cycle and between environmental factors and capabilities in a dynamic perspective. This is actually an aim that we would like to pursue in the future application of our analysis, but even if we cannot trace now a theoretically consistent net of relationship between perspective capabilities, actual capabilities and functionings we are aware of the importance that for instance lack of social interaction can play in the developing of other capabilities and in their realizations for the child that we observe now.

Section 1.1 A list of capabilities for children

In order to conceptualise children well being with a capability approach we follow the procedure suggested by I. Robeyns (Robeyns 2003).

In her paper Robeyns provides an important methodology to endorse a list of capabilities in order to assess gender inequality in Western countries. She provides an exhaustive list of criteria to be implemented in order to endorse a list of capabilities in the case of gender inequality. Robeyns' selection criteria are the following:

1. Explicit formulation: "the list should be explicit, discussed, and defended" (Robeyns 2003 pp70)
2. Methodological justification: in drawing a list we should justify the method used.
3. Sensitivity to contest: the level of abstraction of the list should be appropriate for the aim of the research.
4. Different levels of generality: there are 2 levels: the first is the ideal list that fulfils the above criteria; the second is the empirically implementable list.
5. Exhaustion and non reduction: the list of capabilities should include all important elements.

In this paper we argue that also in the case of children well being, it is important to use that methodology in order to provide a general frame where the specific measures of child well being can be set against.

[5] Waldfogel (2002,p.530) refers to literature showing that 'children from families that are economically disadvantaged gain more from child care in terms of their cognitive development that do other children' and also to NICHD study in the USA that shows that 'infants whose parents had more sensitive childrearing styles fared better than other children in early child care'.

The criterium of explicit formulation.

1. Life and physical health; being able to have good health, not dying prematurely, to be adequately nourished; to have adequate shelter.
   This capability is very much related to family income (and the possibility of having private health care) and to the quality of the national health service across regions,[6]
   In Western society in general the problem is not malnutrition or stunting but unhealthy nutritional habits leading to obesity and overweight.

2. Mental Well being: the capability to have a mental health can be applied to children. Do children have adequate psychological support when needed? Is children psychological development taken into account in the schools?

3. Bodily Integrity. "Being able to move freely from place to place; to be secure against violent assault, including sexual assault and domestic violence."

   The capability of moving freely in the territory is often denied to Italian children, specially to those who live in towns: there has been an increasing concern among parents to let children go to school alone, to play in courtyards and in playgrounds by themselves. This is due to the increase of criminality rates. Security for children in terms of domestic violence and sexual assault are also important issues.
   Sexual and domestic violence are also an important capability for children. Beating children is not so morally condemned in Italy as it is in Northern European countries.
   In the above issues there is a gender components that we need to take into account.

4. Social interaction: Is the child able to interact with other people? What people does the child interact with? Italy is characterized by one of the lowest fertility rate in the world and the probability to observe a child interacting with sisters or brothers or with cousins is not so high. How are the functionings connected to this capability connected to different (extended) family composition, or with the possibility to attend schools or

[6] The latter has a great degree of variation across regions, a signal of different qualities in the health service is the observed mobility of patients across regions in Italian hospitals. In a system of indicators affecting this capability one should include data on quality of national health service, presence of special services devoted to children and on the ratio between pediatrician/children living in one region.

kindergarten.[7] How is the possibility of interacting with parents affected by their labour market condition, by their time constraints, by their educational level, wishing and capability to interact with their children.

5. Education and knowledge: being able to be educated and to use and produce knowledge. This is a basic capability for the development of children. How does the quality of Italian schools differ across the regions? The Innocenti report (Istituto degli Innocenti, 2002) shows how the availability of kindergartens varies across regions. The drop off rates in some regions is also very high for teenagers.

6. Leisure activities, play. The role of this capability in children's well being is essential, however even its functionings are not easily observable. We can look for estimates of consumption of toys by using expenditure surveys, or look for time use data. Asking how this capability can be affected by the environment we could look for data on the presence of schools or play centres for children in different age groups, or of laboratories dedicated to children (painting, music, dance…) and on their attendance and let these data interact with available individual data on the actual leisure activities of children. Pychologists could stress that it is not only important to assess how much the child plays but what types of plays and activities the child does and with whom does he/she play. This capability is strongly related to other children's capabilities lilke social interaction and education. Not playing alone requires interaction with other children, parents or with other individuals who are wishing to interact. Taking Italy we could try to assess how the functionings connected to this capability differ across regions characterised by different presence of schools and organized leisure activities. One could recognize that pure leisure time not devoted to attending laboratories or activities structured by adults is reducing for Italian children and analyse also how new types of playing activities currently more spread amongst them (like videogames) can affect other capabilities also in perspective.
Another important issue is to what extent the child is free from paid or un-paid work. There has been an increased concern for the amount of work performed by children.

<u>The criterium of methodological justification</u>

Methodologically, we have taken into consideration the 4 different lists of capabilities reassumed by Robeyns (2003) in table 1, and selecting the ones that are relevant for children well being. We have excluded capabilities concerned with political freedoms, religion, paid work, time autonomy.

[7] According to Istituto degli Innocenti (2002) in year 2000 only 7,4 places were available for every 100 children aged from 0-2 in Italy, and this percentage is the result of very different situations across Italy from a minimum of 1,9% in Calabria to 18,3% in Emilia Romagna. Also the possibility of attending educational services 'integrativi' that can offer social interaction possibilities for children is differently spread across Italy as Istituto degli Innocenti (2002) shows.

The list provided in this paper is very provisional and it needs to be revised with the help of sociologists and psychologists.

The criterium of sensitivity to context.

The list of capabilities provided above is drawn thinking to the possible applications in social sciences; when a particular measure to promote children well being is introduced (for instance a change in the school system), it is relevant to understand both the effect of that measure on a single indicator of child well being (for instance educational results) and on the compounded measure of child well being.

The criterium of different levels of generality.

Ideally which kind of data set would be needed?
As we have stated in the first part of section 1, measuring children capabilities would require to observe children development during their childhood. The ideal data set would therefore follow children from their birth until they become adults (panel data).
In fact measuring children well being is age dependent. Many functionings can only be measured at a late stage of the child development .
We would also need detailed information for each child, family background, income, and a detailed questionnaire for each of the above–specified capabilities and functionings. In addition to this we would need to distinguish between achieved functionings and capabilities. To this extent it would be important to have information on the social environment the child leaves in. Regional or cities data on the quality of the schooling system, the availability of green area, sports facilities, public subsides to children recreational activities. These data would help in measuring capabilities *vis-à-vis* functionings.

The data used for the empirical application of this paper are specified in Section 3.2.

The criterium of exhaustion and non-reduction.

The list of capabilities provided above includes all important elements for measuring capabilities of Italian children and cannot be reduced.
In the empirical part of this paper we will only attempt to measure one of the capability of the list, the social interaction.

In the applied part of this paper we will try to propose a system of observable measures of functionings and environmental indicators relevant for the development or being a realization of the children's capability of social interaction.

Section 2 To what extent fuzzy set theory and structural equation modelling can measure functionings?

Section 2.1 The fuzzy model: a fuzzy expert system.

Conventional mathematics enables processing of precise information. However, in the reality, we very often meet with imprecise information such as: sufficient well-being, good level of quality of life, etc. People have used imprecise information for thousands of years. However, until quite recently it has not been used at all in methods based on conventional mathematics. Therefore it has been lost. Because of this, the efficiency of many control, modeling, forecasting and decision- making methods was considerably limited all the more, as in some systems imprecise information is the only accessible one.

The domain of mathematics dealing with imprecise information was named Fuzzy Set Theory. This theory in connection with conventional mathematics enables the processing and use of any information.

Since their inception in the mid 1960s by Professor Lotfi A. Zadeh, from the University of California, (Zadeh L.A. (1965)), Fuzzy Set Theory have triggered mixed feeling in scientific community. On one hand, a growing number of devotees have recognized potentials of fuzzy sets to model and solve many real problems, on the other a considerable number of opponents have fiercely fought against these emerging tools. One of their arguments has been a lack of applications.

The situation had change since the mid of 1980s when the so-called "fuzzy boom" occurred primarily in Japan, but then also in Korea and Europe. Basically, the turning point was the launching on the market of fuzzy logic control based appliances and other equipment exemplified by subway trains, cranes, elevators, etc.

In the same period engineering control applications were faced with a first try to use traditional mathematics models for incomplete information problems, such as "Expert Systems". An Expert System is an intelligent machine that uses knowledge and inference procedures to solve control problems that have a so high level of complexity to require significant human expertise for their solutions. The knowledge of an expert system consists of facts and heuristics. The facts usually constitute a body of information that is widely shared, publicly available, and generally agreed upon by experts in the field. Heuristics concerns mostly private information and rules of good judgment that characterize expert-level decision making in the field. These instruments have had a great relevance in engineering applications, but with the increase of problems complexity they show their limits and were abandoned.

The Japanese successful applications of fuzzy logic control, were obtained by a "Fuzzy Evolution" of "Expert Systems", called "Fuzzy Expert Systems", (Von Altrock, C. (1997)).

A Fuzzy Expert System utilizes fuzzy sets and fuzzy logic to overcome some of the problems that occur when the data provided by the user are vague or incomplete. The power of fuzzy set theory comes from the ability to describe linguistically a particular phenomenon or process, and then to represent that description with a small number of very flexible rules. In a Fuzzy Expert System, the knowledge is contained both in its rules and in fuzzy sets, which hold general description of the properties of the phenomenon under consideration. One of the major differences between a Fuzzy Expert System and another Expert System is that the first can infer multiple conclusions. In fact it provides all possible solutions whose truth is above a certain threshold, and the user or the application program can then choose the appropriate solution depending on the particular situation. This fact adds flexibility to the system and makes it more powerful. Fuzzy Expert Systems use fuzzy data, fuzzy rules, and fuzzy inference, in addition to the standard ones implemented in the ordinary Expert Systems.

Functionally a fuzzy system can be described as a function approximator. More specifically it aims at performing an approximate implementation of an unknown mapping $f : A \subseteq R^n \to R^m$ where A is a compact of R^n. By means of variable knowledge relevant to the unknown mapping [Kosko, 1992] and [Wang, 1992] independently proved that fuzzy systems are dense in the space of continuous functions on a compact domain and therefore can approximate arbitrarily well any continuous function on a compact domain. The following are the main phases of a Fuzzy Expert System design:

1. Identification of the problem and choice of the type of Fuzzy Expert System, which best suits the problem requirement. A modular system can be designed. It consists of several fuzzy modules linked together. A modular approach may greatly simplify the design of the whole system, dramatically reducing its complexity and making it more comprehensible.
2. Definition of input and output variables, their linguistic attributes (fuzzy values) and their membership function (fuzzification of input and output).
3. Definition of the set of heuristic fuzzy rules. (IF-THEN rules).
4. Choice of the fuzzy inference method (selection of aggregation operators for precondition and conclusion).
5. Translation of the fuzzy output in a crisp value (defuzzification methods).
6. Test of the fuzzy system prototype, drawing of the goal function between input and output fuzzy variables, change of membership functions and fuzzy rules if necessary, tuning of the fuzzy system, validation of results.

In building Fuzzy Expert System, the crucial steps are the fuzzification and the construction of blocks of fuzzy rules. These steps can be handled in two different ways. The first is accomplished by using information obtained through interviews to the experts of the problem. The second is accomplished by using methods of machine-learning, neural networks and genetic algorithms to learn membership functions and fuzzy rules. The two approaches are quite different. The first does not use the past history of the problem, but it relies on the experience of experts who have worked in the field for years. The second is based on past data and project into the future the same structure of the past.

We can formalize the steps in the following manner. For each linguistic variable, input x_i *(i=1…m)* and output *y*, we have to fix its own range of variability U_i and V. $\forall i$, (*i=1…m*), if n_i is the number of the linguistic attribute of the variable x_i and $\hat{n} = \max_{i\in[1,m]} n_i$, we define the sets

$$A^i = \{A^i_1, A^i_2, .., A^i_{j_i}, .., A^i_{n_i}\},\ B = \{B_1, B_2, ..., B_k, ..., B_r\}$$

where $\forall\ j_i \in [1,\ n_i],\ \forall\ n_i\ \in [1, \hat{n}]$ $A^i_{j_i}$ are the fuzzy numbers describing the linguistic attributes of the input variable x_i, and $\forall k \in [1, r]$, B_k are the fuzzy numbers describing the linguistic attributes of the output variable *y*.

At every elements of A^i and *B* a membership function is associated such that

$$m_{A^i_{j_i}}(x) : U_i \to [0,1] \qquad \text{and} \qquad m_{B_k} : V \to [0,1]$$

The elements of A^i and *B* overlap in some "grey" zone, which cannot be characterised precisely. Many phenomena in the world do not fall clearly into one crisp category or another. Experts, that use abstraction as a way of simplifying the problem, can contribute to identify these "grey" zones.

The choice of the slopes of the elements of A^i and *B* is a mathematical translation of what the experts think about the single terms.
The second step is the block-rules construction.
We define the set of L fuzzy rules, where

$$L \le \prod_1^m n_i\ ,\ \forall\ j_i \in [1,\ n_i],\ \forall\ n_i\ \in [1, \hat{n}]\ \forall k \in [1, r]$$

IF (x_1 is $A^1_{j_1}$) $\otimes$ (x_2 is $A^2_{j_2}$) $\otimes \ldots \otimes$ (x_m is $A^m_{j_m}$) (2.1-1)

THEN (y is B_k), (2.1-2)

The relation (2.1-1) is called "precondition" and the symbol $\otimes$ represents one of the possible aggregation operators. In practical applications, the MIN and MAX operators, or a convex combination of them, are widely used and so a "negative" or "positive" compensation will occur between them.
Instead of Min and Max, it is also possible to use other t-norms and s-norms, which represent different ways of linking the "and" with the "or".
The relation (2.1-2) is called conclusion. The aggregation of precondition and conclusion can be made in several ways. The most used are the MAX and the BSUM methods. The choice depends on the type of application. The MAX has the meaning of keeping as "winner" the strongest rule, in the sense that if a rule is "firing" (activated) more than one time, the result is the maximum level of firing. In the BSUM case, all the firing degree is considered and the final result is the sum of the different level of activation (not over one). In any case, the two methods produce a fuzzy set, which has membership function $m_{agg}(y)$.

Now we have a result of the fuzzy inference system, which is a fuzzy replay. We need to return to a "crisp" value, and this step is called "defuzzification". This operation produces a "crisp" action $\bar{y}$ that adequately represents the membership function $m_{agg}(y)$. There is no unique way to perform this operation. To select the proper method, it is necessary to understand the linguistic meaning that underlies the defuzzification process. Two of these different linguistic meanings are of practical importance: the "*best compromise*" and the "*most plausible result*". A method of the first type is the Centre of Area (CoA) that produces the abscissa of the centre of gravity of the fuzzy output set

$$\bar{y} = \frac{\int_V y m_{agg}(y) dy}{\int_V m_{agg}(y) dy}$$

A method of the second type is the "Mean of Maximum" (MoM). Rather then balancing out the different inference results, this method selects the typical value of the terms that is most valid [ ].

Several authors (Chiappero, 2000, Lelli, 2001, Cheli-Lemmi, 1995) have faced several economic problems using methodological tools based on fuzzy set theory. Their approach is really different from the one we propose in this first paper. The unique concept the two approaches have in common is the necessity to use fuzzy logic and not Boolean logic. The more relevant differences are two. One is due to the starting point.

These authors start from data they have to pass from a crisp definition to a fuzzy one of the several concepts they study (functioning, capability, personal and social characteristics, etc.). Starting with distribution functions they built the membership functions they need. Next they propose different ways to aggregate these results to reach the final evaluation.

Here we propose a completely different method. The basic idea is that this problem is configurable as a multicriteria-problem. The last ones are faceable with the techniques of Knowledge-Based Systems that is starting from knowledge that is: experts. The experts describe which are the initial variables (Var.), how they may be aggregated to have intermediate variables (Int.) and then how to aggregate the last ones to reach the final evaluation (output). At the several levels, they use only linguistic attributes and linguistic rules to aggregate the starting points. They use imprecise information and the process is done without the knowledge of data. It is a procedure completely torn off data. The reasoning followed is peculiar of the problem and has not to be affected by disposable data. They propose clear linguistic attributes and rules. They may be criticized, but we cannot say that the choices they make are not transparent. When the system is ready, a sensitive analysis can be done to control if there are some incongruities. The data are used only at the final step. The data file is put in the system and the output is ready in a second. The choice of experts depends on the type of problem we face. In this particular case we think that the economists are to be supported by psychologists of evolutive age, sociologists and pedagogists and we will only frame for explicative aims the fuzzy scheme followed.

Another important difference is from the mathematical point of view. The unknown function which connects variables with output is, usually, not linear. Even in case of high non-linearity, these methods are able to approximate it very well, as we have said before (paragraph 2.1). The mathematical method other authors propose are average, weighted average and so on. These methods produce linear functions, but, unfortunately, the real world is too much complex to be linear.

2.2 Structural equation modelling

The existence of multiple, inter-related indicators to measure Children's Well being raises the question of how to combine them in empirical research. The MIMIC model is one approach to this problem.
Confronted with the problem of determining the impact of *causes* of child well being, the most basic strategy is to choose a single indicator we believe is the closest (teenager's pregnancy for example) to the unobserved construct (child well being), and ignore both measurement error and information on the remaining indicators.

Alternately we could use the information in all indicators by creating a synthetic variable, such as a simple mean indicator. Based upon a set of casual factors the resulting Ordinary Least Squares model represents perhaps the most restrictive model given the neglect of measurement error, the reduction of an m x 1 vector of indicators to a scalar quantity.
In the MIMIC approach we assume that each of the indicators is a component of child well being; and child well being is an unobserved variable that is linked to the observable indicators.

The principal advantage of this approach is that it does not rely on exact measurement of child well being. Each indicator represents a noisy signal of child well being in a specific age interval.

This modeling strategy has been extensively used in psychometrics and more recently in econometrics (see for example Raiser, M., Di Tommaso, M.L, and Weeks M. 2000), and is founded upon the specification of a system of equations which specify the relationship between a set of unobservable latent variables, a set of observable endogenous indicators and a set of observable exogenous variables.

This approach builds upon the early work of Joreskog and Goldeberger (1975) and Zellner (1970) and has been formalized in the LISREL (Linear Structural Relationships) model of a set of linear structural equations.

Excellent review of the literature is to be found in Bentler and Weeks (1980) and Aignes, Hsiao, Kapteyn, and Wansbeek (1984). The Multiple Indicators and Multiple Causes (MIMIC) approach allows us to think of this model as comprising two parts: a structural equation for children well being and a measurement equation that takes into account that there is no single variable called well-being.

This approach allow us to have an estimate of the influence of a variety of variables both monetary and non-monetary on child well being at different age intervals.

Section 2.2.1 The MIMIC Approach

In examining the relative merits of our modelling strategy we first introduce notation. We let

$Y^O = \left(Y_1^O, Y_2^O, Y_3^O, \ldots\ldots\ldots.Y_m^O\right)'$
and $Y^C = \left(Y_1^C, Y_2^C, Y_3^C, \ldots\ldots\ldots.Y_m^C\right)'$

denote, respectevily $m \times 1$ vectors of ordinal and continuous indicators; $Y_i^O \in \{1, \ldots.., v\} i = 1, \ldots\ldots..m,$ where v represents the number of ordered categories and Y^C represents the latent counterparts to $Y^O = t\left(Y^C\right)$ where $t(.)$ denotes the one to one mapping between the vector of latent variables and the ordinal indicators. We will use Y^* to denote an unobserved latent construct.

Our argument for choosing the MIMIC specification rests upon the belief that the parameters, which are delivered by this approach represent the fundamental objects of interest. In *single* indicator models, each observed measure, here elements of the vector Y^O , is considered a single indicator of a matching unobserved construct, elements of Y^C, such that the moments of interest can be written as $E(Y^O = t\left(Y^C \mid x\right)$.

In contrast multiple indicator models (Muthen 1979)) link multiple observed measures to a reduced dimension of underlying latent variables. In this instance a single indicator model is not appropriate since the moments we wish to estimate are of the form $E(Y^* \mid Y^O = t\left(Y^C, x\right)$. rather than $E(Y^O = t\left(Y^C \mid x\right)$.

By focussing upon the distinction between these two sets of moments, we can show the principal differences between a MIMIC specification and more standard regression-based approaches.

First we note that both MIMIC specification and more standard regression-based approaches (for example, multivariate probit analogues of SUR models[8]) utilise information in all the m indicators. However, the MIMIC model proposed

[8] Multivariate extensions of the binary probit models date from the seminal bivariate probit model first introduced by Ashford and Sowden(1970). For recent examples see Bock and Gibbons (1996), and Gibbons (1996), and Golob and Regan (1998).

here [9] presupposes the existence of two measurement equations: an inner equation where each qualitative indicator, say Y_j^O is linked to its corresponding continuos counterpart Y_j^C; and an outer equation, the standard factor model, providing a mapping from the multiple indicators collected in Y^O to a single unobserved latent construct, Y^*. In contrast discrete versions of SUR, although admitting dependence across observed indicators, are single indicator models,and as a result, do not entertain the existence of, for example, an underlying common factor such as well being.

In this respect the parameters estimated from the set of moments $E(Y^O = t(Y^C \mid x)$ are not the fundamental parameters of interest[10]. In our application we are not interested in the impact of individual characteristics on each of the dimensions of Well Being.

We have few priors on which to base hypothesis testing with respect to individual dimensions.

We construct a system of equations, which specify the relationship between a set of unobservable latent variables Y^*, a set of observable endogeneous ordinal indicators Y^O, and a set of observable exogenoeus variables X .

Section 2.2.2 Model Specification

The structure of the model is as folows:

$$Y = \Lambda^Y y^* + e,\ j = 1,........,m$$

where

$$Y = (Y_1\ , Y_2\ , Y_3\ ,...........Y_m)' \qquad (1)$$

[9] Given the ordinal nature of our observed measures, the form of the MIMIC model proposed here is nonlinear and hs been referred to by Wansbeek and Meijer (2000) as the LISCOMP model.

[10] A welcome by-product of the MIMIC approach is that instead of estimating m regression equations for the set of indicators, we estimate the parameters of a single structural equation. Ignoring covariance terms, and assuming that **x** represents a (s x 1)vector of causes, we have a total of m + s estimable parameters. This compares with a total of m x s parameters and estimate a system of equations over the m indicators

is a $m \times 1$ vector with each element representing an independent indicator of children well being, denoted Y^*. $\Lambda^Y = \{\Lambda^Y{}_1, \Lambda^Y{}_2, \Lambda^Y{}_3 .....\Lambda^Y{}_j\}'$ denotes a $m \times 1$ parameter vector of factor loadings, with each element representing the expected change in the respective indicators following a one unit change in the latent variable. e is a $m \times 1$ vector of measurement errors, with Θ_e denote the covariance matrix.

In addition we posit that children's Well Being is linearly determined by a vector of observable exogeneous variables $x = (x_1, x_2, ...x_s,)'$ and by a stochastic error V giving,

$$Y^* = x'g + V \qquad (2)$$

where g is a $s \times 1$ vector of parameters.

Examining (1) and (2) we may think of our model as comprised of two parts: (2) is the structural (or state) equation and (1) is the measurement equation reflecting that the observed measurements are imperfect indicators. The structural equation specifies the casual relationship between the observed exogeneous causes and children well being. Since Y^* is unobserved, it is not possible to recover direct estimates of the structural parameters g. Combining (1) and (2) the reduced form representation is written as

$$y = px + v$$

where $p = \Lambda^Y g'$ is the $m \times s$ reduced form coefficient matrix and $v = \Lambda^Y V + e$ is the reduced form disturbance.

For identification issues and treatment of ordinal indicators see Raiser, M., Di Tommaso, M.L, and Weeks M. 2000

Section 3 – An application to the capability of social interaction.

Section 3.1 Building a System of indicators to measure the Capability of Social Interaction

As stated in the first part of this paper the capability of social interaction and its functioning can be affected by several environmental factors that we will try to highlight in this section aiming also at an application to Italian data.

The very possibility to interact depends on the presence of other individual in the environment where the child lives, therefore it is important to have data at aggregate level on fertility rates and on average number of children in the area where the child lives and it would be better to have access to individual data allowing to get precise information on the household's composition of the child, as well as on the employment condition of parents and their education. One should be aware that the same individual characteristic may affect social interaction capability in different ways. For instance a parent being manager may have only a limited amount of time to devote to interacting with the child (this should have a negative effect on functioning parent's interaction with child) but can increase family income and thus can make affordable private childcare services or access to children laboratories or to a bundle of commodities and services that may allow the child a higher level of interaction with other children or adults.

One should also analyse the impact of institutional factors (laws, collective agreements) on the employed parents' possibility to interact with their child. On this regards Law 53/2000 in Italy made it possible for parents to use parental leave up to the child's age of 8. By using ISTAT 2002-2003 survey on time use, ISTAT (2004) shows that if more than 2/5 of employee working women used this possibility amongst the 44,000 working mothers who did not use the leave 5% (especially blue collars) stated that they were not allowed to use it.

Having access to extended family composition data could provide information also on the probability of having cousins and other relatives that the child can meet (therefore data on how often does the child meet relatives of different age is useful).

Another area where the child can experience social interaction is at school or kindergarten and it becomes significant to test at each age level the diffusion of child care services and schools of different types in the area where the child lives. One should notice that the presence of childcare services for children under 3 years old is particularly low in Italy and not evenly distributed across the different regions as Tab.1 and 2 from Istituto degli Innocenti (2002) clearly show. Also full-time school is not evenly distributed across Italy, nor the possibility of having lunch at school with other school mates.

Tab. 1 Percentage of places available in kindergarten (private and public) for region in Italy on children aged 0-2 living in each region

	%
Piemonte	10,7
Valle d'Aosta	12,3
Lombardia	9,7
Trentino Alto Adige	7,5
Veneto	7,2
Friuli Venezia Giulia	7,8
Liguria	9,7
Emilia Romagna	18,3
Toscana	11,3
Umbria	11,6
Marche	11,5
Lazio	8,5
Abruzzo	4,1
Molise	2,9
Campania	2,2
Puglia	2,7
Basilicata	5,2
Calabria	1,9
Sicilia	4,7
Sardegna	6,4
Italy	7,4

Source Istituto degli Innocenti (2002) tav.3 p.17

Tab. 2 Percentage of places in public kindergarten available for region in Italy on children aged 0-2 living in each region

	%
Piemonte	9,7
Valle d'Aosta	12,3
Lombardia	8,9
Bolzano	3
Trento	11
Veneto	5,5
Friuli Venezia Giulia	6,1
Liguria	9,3
Emilia Romagna	17,4
Toscana	10,7
Umbria	10,6
Marche	9,7
Lazio	7,5
Abruzzo	3,8
Molise	2,5
Campania	1
Puglia	2,1
Basilicata	4,4
Calabria	1,1
Sicilia	4,7
Sardegna	5,7
Italy	6,5

Source Istituto degli Innocenti (2002)

New types of services are spreading in Italy for children in preschool age and could be particularly useful to increase social interaction with other children especially for those who could not have access to kindergarten, they are unevenly present in different regions and only 0,62% of children aged less than 3 do attend them on average in Italy.

It could be useful to have data also on quality of childcare,[11] since time spent in high quality childcare services has been found to have a positive impact on child's social competence an cooperation later in his life and on peer relations.[12] The capability of social interaction can be realized by interacting with other children during courses or laboratories or in open spaces (therefore it becomes relevant at an aggregate level to measure the supply of laboratories or societies caring for the diffusion of these activities and the presence of parks in the areas

[11] On the indicators of quality of childcare see Waldfogel (2002). As she stresses: '…there is no consensus on how to define quality of child care. Child care advocates tend to point to structural features of child care programs such as the group size, child-staff ratio, and health and safety requirements, while parents tend to look for a caregiver who is warm and sensitive, and conveniently located. Researchers try to measure both types of characteristics, as well as continuity and stability of care' (Waldfogel, 2002, p.544).

[12] These effects have been found in different studies based on US and UK data referred to by Waldfogel (2002 pp.540-541).

where the child lives, whereas at micro level it is important to measure the attendance of children of these spaces and activities).
Measuring functionings of this capability involves having individual data on how often the child meets other children, relatives or other people and it is important to see what type of activities they do together (for instance spending 3 hours a day with a brother watching television or making videogames is different than having a football match in the open air). In a gender perspective it is also important to see how often a child meets persons of different sex.
We will try to make an application to measuring social capability for children aged from 6 to 10 living in Italy by introducing the flow in Figure 1 and by implementing it by means of different models in the following sections.

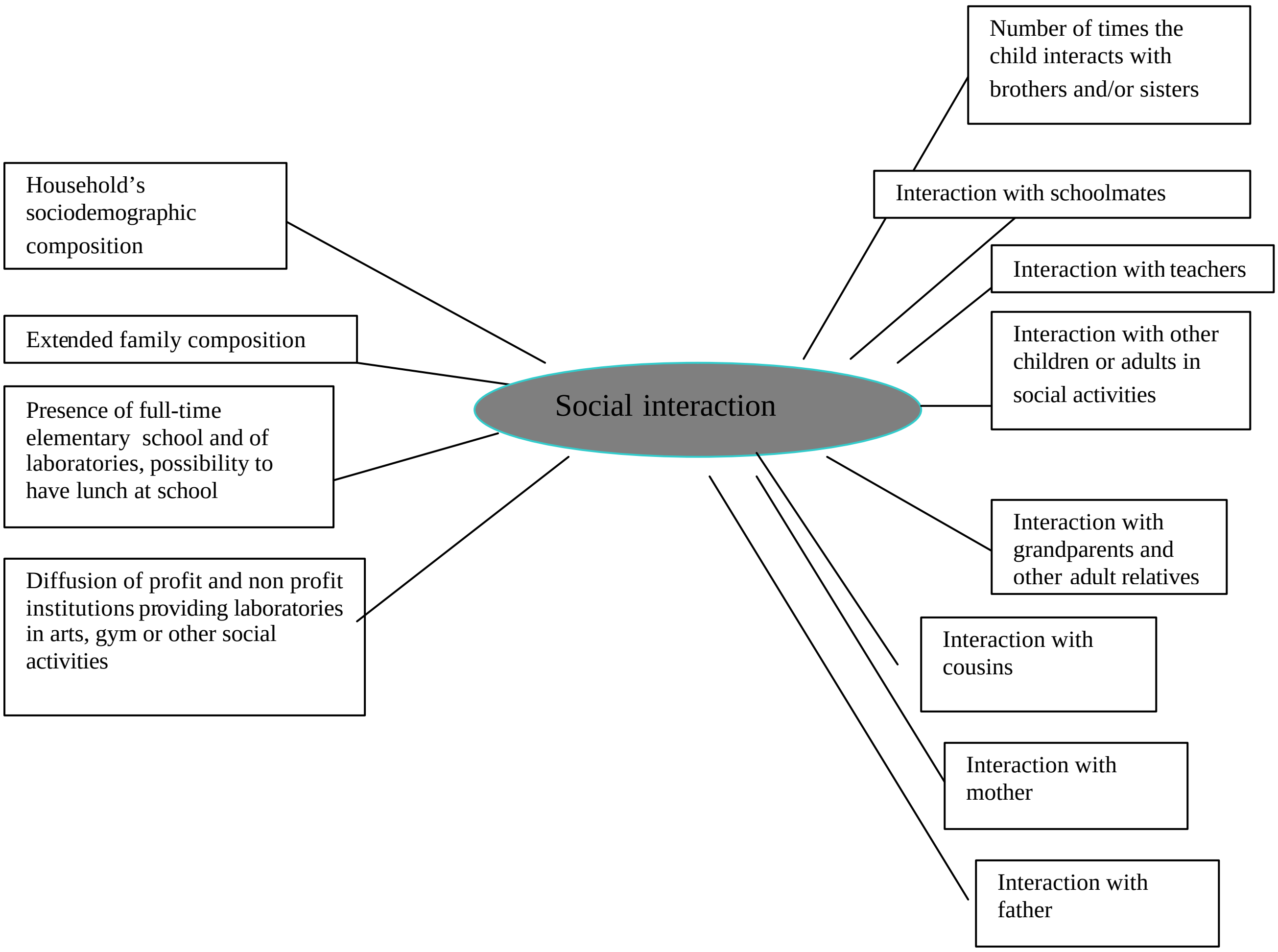

Fig.1 Social interaction, functionings and environmental factors.

This flow on the left hand side assumes environmental factors affecting capability of social interaction as well as its conversion in functionings which are listed in the right hand side of the flow.

Section 3.2 Building a Fuzzy set model to measure the Capability of Social Interaction

In this section we will try to apply the logical system of indicators presented in Section 3.1 by looking at useful data and by proposing a fuzzy expert scheme to measure them. The main source of data that we will use is ISTAT (Italian Institute for Statistics) 1998 *Famiglia, soggetti sociali e condizione dell'infanzia* survey on about 24,000 households and about 60,000 individuals that contains together with useful information on each child's family composition (though not household's income) also information on each child's variables (sex, age, level and type of education) and on each child's activities with relatives and other individuals.

The proposed scheme can disaggregate the variables affecting social interaction capability as following:

Individual information affecting social interaction:

- Household socio-demographic composition:
    - Number of member and age
    - Household type: one earner, double earner, lone parent
    - Parents' education level and their employment condition
- Extended household composition: presence of grandparents and number of cousins
- Attendance of full-time elementary school (hours at school)
- Possibility to have lunch at school
- Hours spent in front of the television (this can limit the total amount of time devoting to other activities and to interact with others)
- Whether the child has experienced during last year any violent act from other children of the same age
- How often does the child play in the open air

Aggregate data on the presence of full time elementary school, or on the attendance of laboratories in the region[13] where the child lives can be obtained by ISTAT multipurpose survey taking into account also the size of town where the family lives.

Observed functionings can be aggregated into:

- Interaction with brother or sisters (whether the child sleep alone or with brother/sister; whether the child returns or go to school with brother or sister, whether the child plays with brother or sisters)
- Interaction with schoolmates (whether the child plays with schoolmates)
- Interaction with cousins (whether the child meets them once or more than once a month)

[13] The survey is statistically significant at regional level.

- Interaction with friends (whether the child returns or go to school with friends, whether the child plays with friends meeting them outside structured activities and how often does the child meet them, whether the child meet more children of his/her sex, whether the child participates at parties)
- Interaction with grandparents (whether the child returns or go to school with grandparents, whether the child is cared by his/her grandparents while not at school, whether the child plays with grandparents)
- Interaction with mother (whether the child returns or go to school with her, activities that the child does with her, how often does the child play with her, whether the child sleeps with her)
- Interaction with father (whether the child returns or go to school with him, activities that the child does with him, how often does the child play with him, whether the child sleeps with him)
- Interaction with other in cultural, environmental or 'boy scouts' societies (whether the child participates and how often)
- Interaction with other children or adults in social structured activities (gym, music, arts…): whether the child attends courses (paid or unpaid) how many times a week (the survey provide us information on the number of hours devoted by each child to each course)

By using fuzzy set we can measure each observed functioning (by aggregating different indicators of the same functioning according to rule that will be made explicit) and then aggregate the functionings to have an estimate of the unobserved child's capability of social interaction.

To make a first attempt we can apply fuzzy expert system to two functionings: interaction with father and interaction with other children or adults in social structured activities (gym, music, arts…):

<u>Interaction with father</u>

For this functioning we can observe:
How often does he play:
If he plays with the father every day or sometimes during the week is good for social interaction, if he plays once a week is not so bad, if he plays less than 4 times a month or a few times in the year is bad and if he never plays is very bad.
Types of games made more frequently with father:
Bad if videogames, good if others (games of role, puzzles, in gender perspective it can be very good if they play with the father by performing housework activities cooking and so on)

Observed variables affecting the conversion of social interaction in this specific functioning are:
- Father's presence
- Father's education level
- Father's employment condition

Interaction with other children or adults in social structured activities

For this functioning we can observe whether child attends courses (paid or unpaid) and how many times a week (the survey provide us information on the number of hours devoted by each child to each course)

Observed variables affecting this functioning:
This variable is affected by family income and by the public and private (profit and non profit) supply of these courses that one should try to measure on average for each region.

Here we propose the Fuzzy scheme of "Interaction with father" (output)

For this functioning we can say that the variables affecting the conversion of social interaction in this specific functioning are:

- Father's presence (var1)
- Father's education level (var2)
- Father's employment condition (var3)

  How often does he play (Var4)

  Types of games made more frequently with father (var5).

The first three variables are aggregated together in an intermediate variable (int1) we may call "Father".
Then Int1, var4 and var5 are aggregated together to obtain the final otuput as in Fig.2.

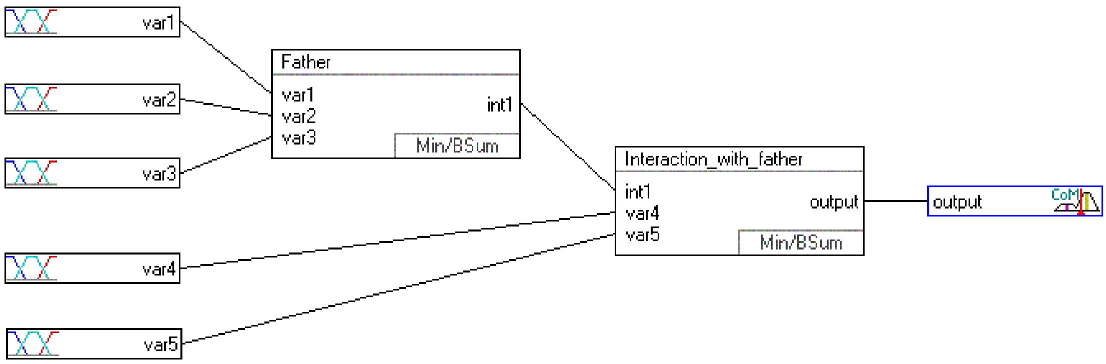

Fig. 2 Interaction with father – a fuzzy representation

We may begin with three linguistic attributes for every variable1-5: Low, Medium, High.
As example a way to fuzzify the var4 may be this type of judgement:
If he/she plays with the father every day or sometimes during the week is good for social interaction, if he plays once a week is not so bad, if he plays less than 4 times a month or a few times in the year is bad.
A way to fuzzify the var5 may be this type of judgement:

Bad if videogames, good if others (games of role, puzzles), in gender perspective it can be very good if they play with the father by performing housework activities cooking and so on.
The intermediate variable Int1 has usually a more detailed type of judgement with five linguistic attributes: Very Low, Low, Medium, High, Very High. Combining the three observed variables may give rise to an outcome of the type described in Tab.3.

IF			THEN	
var1	**var2**	**var3**	**DoS**	**int1**
low	low	low	1.00	very_low
low	low	medium	1.00	very_low
low	low	high	1.00	low
low	medium	low	1.00	very_low
low	medium	medium	1.00	low
low	medium	high	1.00	medium
low	high	low	1.00	low
low	high	medium	1.00	medium
low	high	high	1.00	high
medium	low	low	1.00	very_low
medium	low	medium	1.00	low
medium	low	high	1.00	medium
medium	medium	low	1.00	low
medium	medium	medium	1.00	medium
medium	medium	high	1.00	high
medium	high	low	1.00	medium
medium	high	medium	1.00	high
medium	high	high	1.00	very_high
high	low	low	1.00	low
high	low	medium	1.00	medium
high	low	high	1.00	high
high	medium	low	1.00	medium
high	medium	medium	1.00	high
high	medium	high	1.00	very_high
High	high	low	1.00	high
High	high	medium	1.00	very_high
High	high	high	1.00	very_high

Table 3: Rules of the Rule Block "Father"

We can then combine the results of the first block of variables with variables 4 (how often does the father play with child) and 5 (what types of games are more frequently made with father) to get the outcome for ‘interaction with father’.

IF			THEN	
int1	**var4**	**var5**	**DoS**	**output**
very_low	low	low	1.00	very_low
very_low	low	medium	1.00	very_low
very_low	low	high	1.00	very_low
very_low	medium	low	1.00	low
very_low	medium	medium	1.00	low
very_low	medium	high	1.00	low
very_low	high	low	1.00	medium
very_low	high	medium	1.00	medium
very_low	high	high	1.00	medium
low	low	low	1.00	low
low	low	medium	1.00	low
low	low	high	1.00	low
low	medium	low	1.00	medium
low	medium	medium	1.00	medium
low	medium	high	1.00	medium
low	high	low	1.00	medium
low	high	medium	1.00	medium
low	high	high	1.00	medium
medium	low	low	1.00	low
medium	low	medium	1.00	low
medium	low	high	1.00	low
medium	medium	low	1.00	medium
medium	medium	medium	1.00	medium
medium	medium	high	1.00	medium
medium	high	low	1.00	high
medium	high	medium	1.00	high
medium	high	high	1.00	high
high	low	low	1.00	medium
high	low	medium	1.00	medium
high	low	high	1.00	medium
high	medium	low	1.00	medium
high	medium	medium	1.00	medium
high	medium	high	1.00	medium
high	high	low	1.00	high
high	high	medium	1.00	high
high	high	high	1.00	high
very_high	low	low	1.00	medium
very_high	low	medium	1.00	medium
very_high	low	high	1.00	medium
very_high	medium	low	1.00	high
very_high	medium	medium	1.00	high
very_high	medium	high	1.00	high
very_high	high	low	1.00	very_high
very_high	high	medium	1.00	very_high
very_high	high	high	1.00	very_high

Table 4: Rules of the Rule Block "Interaction_with_father"

However this is only a first attempt that we would like to fill with 'sense'. For instance we can have a psychologist's view on how the type of game the father

does with the child can be judged (the rule that we have expressed in this simulation is just based on our personal judgement) or expert's view on the effect of father's employment status or education on the capability of interaction with his child.
Moreover the same environmental variable can have a different impact on different functionings of the same capability. For instance having a manager as father can imply having a father with a limited amount of leisure that he can devote to interaction with his child (negative effect on interaction with father) but can be a proxy of a higher family income that can allow the child to improve his/her social interaction in paid courses.

4. Concluding remarks and future research

This paper explores the possibilities of using fuzzy inference system and structural equation modelling to measure capabilities referred to children's well being both at a theoretical and empirical level.

In this paper we present a first attempt to use a fuzzy expert system (FES) to face the child well being. The necessity to use a fuzzy approach, felt by other authors, here is treated in a new way. We utilize an instrument that is typical of Artificial Intelligence, in which the support of experts, economists and psychologists is a fundamental tool. As this fuzzy support has had a lot of successful applications, not only in engineering world but even in economic and financial ones, (Facchinetti G.- Cosma S.- Mastroleo G.- Ferretti R. (2001), Facchinetti G.- Franci F.-Mastroleo G.- Pagliaro V.- Ricci G. (2002), Facchinetti G. -Lalla M. -Mastroleo G. (2003), Facchinetti G.- Mannino I.- Mastroleo G.- Soriani S. (2003), Magni C.A. - Mastroleo G. - Vignola M. - Facchinetti, G. (2004)), we trust that it is a good proposal even in this case and in several others connected, like quality of life, of work and so on. The attempt to apply this method to children's well being is only a preliminary attempt, before filling it with available data, we think it is necessary to enlarge our research group to experts from other disciplines that can provide theoretical reasonings for the rules that we will assume in the fuzzy scheme. Our aim is to insert experts' justified rules in the fuzzy scheme that we are building and to highlight possible interactions with different environmental variables and capabilities leading to child well being in Italy by using available data set. The scheme we are currently working to can allow for simulation of different theoretical views on the builiding of children's capabilities, as well as simulations on how children living in different areas or having different household structures have different well being and different functionings by entering in the same fuzzy set scheme values for functionings referring to children from different areas or different household types currently available.
We will then analyze how the different techniques used to measure child well being differ according to the obtained results and theoretical expectations.

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