The Political Economy of Intergenerational Cooperation

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Abstract

Individual actions coordinated only by the market do not generally yield an efficient allocation of consumption over the life cycle of each generation, and across generations. The existence of a self-enforcing "constitutional" arrangement, delivering an efficient allocation of consumption, but not a socially optimal population profile, can be demonstrated at the family level. A similar arrangement at societal level is less plausible. Voting models recognize that the policy introduced by one parliament can be reversed by the next. The probability that a parliament will vote for transfers to children, as well as to the old, increases if the former serve to pay for education, since that raises the contributive capacity of tomorrow’s adults, and will thus benefit tomorrow’s old people.

1 Introduction

"Let us assume that men enter the labor market at about the age of twenty. They work for forty-five years or so and then live for fifteen years in retirement. Naturally, ... men will want to consume less than they produce in their working years so that they can consume something in the years when they produce nothing. ...

If there were only Robinson Crusoe, he would hope to put by some durable goods which could be drawn on in his old age. He would, so to speak, want to trade with Mother

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Nature current consumption goods in return for future consumption goods. ... 

For the present purpose, I shall make the extreme assumption that nothing will keep at all. Thus no intertemporal trade with Nature is possible. If Crusoe were alone, he would obviously die at the beginning of his retirement years.

But we live in a world where new generations are always coming along. ... [C]annot men during their productive years give up some of their product to bribe other men to support them in their retirement years?" (Samuelson, 1958)

The answer to Paul Samuelson’s question is clearly yes, if there are ways of ensuring that the bribed person will deliver his side of the deal when the time comes. Samuelson’s own solution is what he calls ”social contrivances”: contract law and its associated legal enforcement apparatus, money that ”gives workers of one epoch a claim on workers of a later epoch” (Samuelson, 1958). But what about children? They need support too. Indeed more because, unlike the old, they have not had an earlier phase of life in which to put by durable goods. Therefore, if anyone is willing to be ”bribed”, that is children. Samuelson’s question could then be usefully re-worded to ”cannot men during their productive years give up some of their product to bribe children to support them in their retirement years?” The answer is again a conditional yes, but the condition is now harder to satisfy, because a child cannot enter into a legally binding engagement, and because a very long time, twenty or more years, will have to elapse before he or she is in a position to honour his supposed debt. Here, Samuelson’s contrivances are not much help.

Why is there no mention of this side of the problem in Samuelson’s analysis? As Martin Shubick perceptively put it, ”... Samuelson’s model is implicitly a three period model where he dropped the first period by the assumption that child support was to be purely instinctive and hence not in the analysis” (Shubick, 1981). The same implicit assumption underlies much of the subsequent literature on the subject, including many of the articles reviewed in this chapter. The basis for making such an assumption, one may suppose, is that successful animal species are genetically programme to care for their offspring. But is that enough? The existence of laws and social norms deputed to ensure that children get adequate support suggests that it is not. In what follows, support for children and for the aged will be treated as two sides of the same coin.

In the simplest analytical framework, individuals derive utility from their lifetime consumption of market goods only. In richer frameworks, they derive utility also from leisure, and from personal services supplied
by their own parents or children (implying that such services do not have perfect market substitutes). Allowing for the former affects second-best policy, but makes remarkably little difference otherwise. Allowing for the latter increases the explanatory power of the model, and broadens the scope for intergenerational cooperation, but the terms of the question remain essentially the same. Parents well disposed towards their children derive utility also from the consumption, or utility, of the latter. The converse is equally true. As Robertson (1956) put it, however, love or altruism is a scarce good that economists should economize. Indeed, the hypothesis that intra-family transfers are systematically generated by altruistic motivations appears to be rejected by the data. To be on the safe side, therefore, we shall look first for the possibility of mutually beneficial arrangements between self-seeking individuals, and then ask what would happen in the presence of altruism.

In much of the literature reviewed here, fertility is endogenous. It is treated as exogenous, for the sake of tractability, only when educational investment is considered, and in political equilibrium models. For simplicity of exposition, all the authors reviewed reason as if reproduction took place by parthenogenesis (but nothing of relevance to our present concerns changes if one allows for sexual reproduction). For this reason alone, rather than for political correctness, we use the feminine gender throughout.

Throughout, we divide the life-cycle of each person into three periods, labelled \( i = 0, 1, 2 \). A person is said to be a child in period 0, an adult in period 1, or old in period 2. Adults are able to produce income, and have children. Children and the old can do neither. We adopt the convention of calling \( t \) the generation that reaches adulthood at date \( t \). As individuals are active in that period only, this has the expositional advantage of making the date of the action coincide with the generational label of the actor. Since we are only interested in intergenerational issues, we shall generally abstract from intra-generational differences. Re-distribution between families will thus be outside the scope of the analysis.

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1See, for example, Peters (1995), Anderberg and Balestrino (2002).
2That is because, in developed countries, transfers from children to parents are largely non-monetary; see, for example, Cigno and Rosati (2000).
4They reason, also, as if children could be produced by fiat, but nothing of substance changes if this assumption is replaced with the more realistic one, that sexual behaviour and contraceptive practice condition the probability distribution of a birth.
2 A normative benchmark

Before embarking on an analysis of the institutions that might make it possible for members of a generation to cooperate with members of another generation, it is useful to establish a normative benchmark against which to measure the performance of any such arrangement. In this section, we approach the issue under the assumption that capital is the only durable good (we shall introduce a second asset, human capital, and deal with the issue of educational investment, in Section 4).

Let $k^t$, $y^t$ and $n^t$ denote, respectively, capital, income and the number of children of each member of generation $t$ (or capital, income and fertility per adult at date $t$). Income is determined by

$$y^t = f(k^t),$$

where $f(.)$ is a per-adult production function. We postulate a small open economy and perfect capital mobility, so that the interest rate, $r^t - 1$, is exogenously determined. The capital stock is then implicitly determined by

$$r^t = f'(k^t),$$

Since income is net of capital depreciation, the resource constraint for any date $t$ may be written as

$$k^t - r^t d^t + f(k^t) = \frac{c_{2}^{t-1}}{n_{t-1}} + c_1^t + (p + c_0^{t+1} + k^{t+1} - d^{t+1}) n^t,$$

where $c_i^t$ is the consumption of a member of generation $t$ in the $i$-th period of life ($i = 0, 1, 2$), $d^t$ the foreign debt per member of generation $t$, and $p$ is a positive constant, representing the unavoidable cost of having a child. Since this constant will include the subsistence part of a child’s consumption, the variable $c_0^{t+1}$ is to be interpreted as the above-subsistence consumption of a child born at date $t$.\(^5\)

2.1 Optimal consumption and fertility in a simple framework

Let the lifetime utility of each member of generation $t$ be given by

$$U^t = u_0(c_0^t) + u_1(c_1^t) + u_2(c_2^t),$$

\(^5\)We could similarly introduce constants representing subsistence consumption in periods 1 and 2 of a person’s life, and define $c_1^t$ as above-subsistence consumption in period 1 by a person born at $t - 1$, $c_2^{t-1}$ as above-subsistence consumption in period 2 by a person born at $t - 2$, but that would serve no useful purpose. By contrast, $p$ needs to be there anyway, because procreation has to have a fixed cost for the fertility choice problem to be bounded.
where \( u_i(\cdot) \) is a concave function, with \( u_i(0) = 0 \), and \( u'_i(0) = \infty \). Suppose that society is interested in maximizing the Millian welfare function

\[
W^0 = \sum_{t=0}^{\infty} (\delta)^t U^t, \quad 0 < \delta \leq 1, \tag{5}
\]

with \( c^0_t \) given, subject to (3) for each \( t \). The first-order conditions for a social optimum may be written as

\[
\frac{u'_1(c^1_t)}{u'_2(c^2_t)} = r_{t+1}, \tag{6}
\]

\[
\frac{u'_0(c^0_t)}{u'_1(c^1_t)} = r^t = \frac{u'_1(c^{t-1}_1)}{u'_2(c^{t-1}_2)} \tag{7}
\]

and

\[
\frac{u'_2(c^2_t) c^2_t}{u'_1(c^1_t) n^t} = p + c^{t+1}_0 + k^{t+1} - d^{t+1}, \tag{8}
\]

for each \( t \).

Equations (6) and (7) are the necessary conditions for a Pareto-optimal allocation of consumption over the life-cycle of each generation, and across generations. Equation (8) tells us that the marginal social benefit and the marginal social cost of fertility (both expressed in terms of current consumption of generation \( t \)) must be equalized. Since the amount that each of the present adults will consume in old age will have to come out of the income produced by the new person, the marginal social benefit of fertility, \( \frac{u'_2(c^2_t) c^2_t}{u'_1(c^1_t) n^t} \), is the current consumption equivalent, for a member of generation \( t \), of the contribution that an additional member of generation \( t + 1 \) would make to her future consumption. The marginal social cost of fertility is the sum of two terms. The first is the cost of raising a child, \( (p + c^{t+1}_0) \). The second \( (k^{t+1} - d^{t+1}) \) is the social cost of equipping the future adult with \( k^{t+1} \) units of capital, net of the foreign debt that this extra person would inherit.

If \( r^t = r \) for all \( t \), the social optimum has a steady state \( (c^t_i = c^*_i \text{ for all } t) \) characterized by

\[
n = \delta r. \tag{9}
\]

\footnote{There is also the constraint that, for each \( t \), \( n^t \) cannot be less than zero, or greater than a certain physiological maximum. In reality, these restrictions may well be binding for particular women, but average fertility is always inside the limits. Since, in our analysis, all women are the same, we follow the common practice of assuming that these restrictions are not binding at the optimum.}
Therefore, in a comparison across steady states, the optimal rate of population growth \((n - 1)\) will be no higher than the given rate of interest \((r - 1)\).

All of this is on the assumption that society is concerned with the utility of the representative member of each generation. If society were interested in aggregate utility, it would maximize the Benthamite welfare function

\[
W^0 = \sum_{t=0}^{\infty} \beta^t N^t U^t, \quad 0 < \beta \leq 1, \tag{10}
\]

where

\[
N^t \equiv \prod_{j=0}^{t} n^{j-1} \tag{11}
\]

is the number of persons in generation \(t\), with \(c_0^1\) and \(n^{-1}\) given. Since the Pareto criterion applies only to comparisons across consumption streams relating to the same population profile \((N^0, N^1, N^2, ...)\), the efficiency conditions would remain (6) and (7). The fertility condition would become

\[
\frac{(\beta)^t W^{t+1}}{u_1'(c_1^t)} + \frac{u_2'(c_2^t) c_2^t}{u_1'(c_1^t) n^t} = p + c_0^{t+1} + k^{t+1} - d^{t+1} \tag{12}
\]

for each \(t\). This differs from (8) in that the social benefit from adding a person to generation \(t\) has an extra term, \(\frac{(\beta)^t W^{t+1}}{u_1'(c_1^t)}\), representing the equivalent, in terms of current consumption for a member of generation \(t\), of the sum of the lifetime utilities of the new member and all her descendents. Given Benthamite social preferences, a steady state exists only if

\[
\delta r = 1. \tag{13}
\]

What this means is that the social optimization problem can have a steady-state solution only if the internationally determined interest rate \((r - 1)\) happens to equal the given rate of social time preference \((1 - \delta)\).

Since that is unduly restrictive, we go for the Millian hypothesis.

### 2.2 Some extensions

In some studies, for example Groezen, Leers and Meijdam (2002), individuals are assumed to derive utility not only from their own consumption, but also from the number of children that they beget. Since
parents do not derive utility from their children’s consumption or utility, this may be interpreted as saying that people are biologically predisposed to reproduce themselves, irrespective of whether their offspring will live prosperous or miserable lives. In other studies, parents derive utility also from each child’s consumption, as in Kollmann (1997), or utility, as in Becker and Barro (1988). Only then can one talk of (descending) altruism. Since all the approaches mentioned are special cases of Becker-Barro, we concentrate on the latter.

Let the utility of generation $t$ be given by

$$U^t = u_0(c_0^t) + u_1(c_1^t) + u_2(c_2^t) + u_3(n^tU^{t+1}), \quad (14)$$

where $u_3(.)$ is another concave function, with the same properties as $u_0(.)$, etc.$^7$ Since a similar expression applies also to generations $t+1$, $t+2$, etc. this implies that the utility of each person ultimately depends on the consumption profile of her entire dynasty. Given (14), the first-order conditions for the maximization of (5), subject to (3), will still include (6) and (7), but fertility must now satisfy

$$\frac{u_3'(n^tU^{t+1})}{u_1'(c_1^t)} U^{t+1} + \frac{u_2'(c_2^t)}{u_1'(c_1^t)} n^t = p + c_0^{t+1} + k^{t+1} - d^{t+1}. \quad (15)$$

Comparing this with (8), we can see that the marginal social benefit of fertility now includes an extra term, $\frac{u_3'(n^tU^{t+1})}{u_1'(c_1^t)} U^{t+1}$, representing the current consumption equivalent of the pleasure that parents derive from having another child. In steady state, (9) must still hold. Therefore, population must still grow at a rate that is no greater than the rate of interest.

3 Institutions

Are there institutions such that a decentralized economy will generate a socially optimal solution, or at least allocate consumption efficiently? The second question refers to a given population profile. The first presupposes a criterion for choosing among different population profile (e.g., the Millian one). We shall start by looking at the efficiency issue under the conventional assumption that individuals interact only through the market, taking it for granted that Samuelson’s “social contrivances” are firmly in place. Then, we shall allow for the possibility that individuals interact also through the family.

$^7$Becker and Barro (1988) use a special form of (14).
3.1 The market

Modigliani’s life-cycle model provides a useful starting point. In this model, everyone is out for himself (children support themselves by borrowing, old people consume their own savings). Fertility, and thus the population profile, is exogenous. Given the interest rate \((r^t - 1)\), the wage rate is determined by

\[
w^t = y^t - (r^t - 1) \frac{k^t}{r^t}.
\]

Each member of generation \(t\) chooses her own consumption stream \((c^t_0, c^t_1, c^t_2)\) so as to maximize (4). Assuming perfect capital markets, this person can choose how much to borrow or lend in each period of her life, subject only to the lifetime budget constraint

\[
(p + c^t_0) r^{t-1} + c^t_1 - w^t + \frac{c^t_2}{r^t} \leq 0.
\]

The solution to the individual optimization problem of generation \(t\) satisfies (6), and the first equation in (7). Since the same is true of generation \(t - 1\), the second equation in (7) is also satisfied.

If the rate of population growth were lower than the rate of interest, the allocation would then be efficient. Since fertility and the rate of interest are exogenous, however, that could only be by chance. Furthermore, markets are far from perfect. In particular, credit in period 0 is likely to be rationed, not only for the well-known difficulties that the young face in borrowing against future earnings (Stiglitz and Weiss, 1981), but also because minors are legally debarred from mortgaging their future. Therefore, the agent faces an additional constraint,

\[
p + c^t_0 \leq b,
\]

where \(b\) is the maximum she can borrow in period 0. If (18) is binding for some \(t\), the allocation is inefficient.\(^8\)

What if fertility is endogenous? If that is the case, the population profile depends on decisions taken by individual members of each generation, and there is nothing to ensure that these decisions will satisfy either (8) or (12). Worse, the economy will not get off the ground. Since a child costs her parent at least \(p\),\(^9\) but yields no benefit, generation \(t\) will in fact choose \(n^t = 0\). Hence, there will be no generation \((t + 1)\). But, if there is no reason for generation \(t\) to produce a generation \((t + 1)\), there is equally no reason why generation \((t - 1)\) should have produced

\(^8\)If \(b < p\), the agent will not live to be an adult.

\(^9\)We are assuming that parents are not allowed (by law, or by some other form of social control) to let their children starve.
a generation \( t \), and so on. In other words, a Modigliani economy is incompatible with fertility choice.

Let us see what would happen in the more favourable situation hypothesized by Becker and Barro (1988), where agents derive direct utility from the consumption streams of all their descendents, as well as from their own. Given perfect markets, generation 0 chooses its own lifetime consumption stream, together with the size and consumption streams of all subsequent generations, so as to maximize

\[
\sum_{t=1}^{\infty} \left( p + c_0^t \right) r^{t-1} + c_1^t - w^t + \frac{c_2^t}{r^t} \leq a^0 + w^0 - c_1^0 - \frac{c_2^0}{r_0^0}. \tag{19}
\]

subject to the dynastic budget constraint,

\[
R^t = \prod_{j=1}^{t} r^{j-1}, \tag{20}
\]

where

\[
a^t \equiv c_1^t - w^t + \frac{c_2^t}{r^t} + \left( p + c_0^{t+1} + \frac{a^{t+1}}{r^t} \right) n^t \geq 0 \tag{21}
\]

for each \( t > 0 \), where \( a^t \) is the amount inherited as an adult by (the assets of) a member of generation \( t \). The first constraint is written on the assumption that adults are not rationed in the credit market. The second set of restrictions arises from the fact that nobody can be obliged to accept an onerous gift.

If (21) were never binding, as Becker and Barro assume, the first-order conditions would yield (6), (7) and

\[
\frac{u_3^t \left( n^t U^{t+1} \right)}{u_1^t \left( c_1^t \right)} U^{t+1} + \frac{u_2^t \left( c_2^t \right)}{u_1^t \left( c_1^t \right)} n^t = p + c_0^{t+1} + \frac{a^{t+1}}{r^t} \tag{22}
\]

for each \( t \). Since, at any date, the assets held by the representative adult equal the country’s net credit position per adult,\(^{10}\)

\[
k^{t+1} - d^{t+1} = \frac{a^{t+1}}{r^t}, \tag{23}
\]

(22) would then be identical to (15), and the laissez faire would coincide with the Millian optimum.\(^{11}\) There is, however, no reason to expect
that (21) is never binding. Given convex preferences, parents will in fact wish they could make negative bequests if their children are sufficiently richer than themselves.\footnote{Recall that any bequest is on top of direct payments for the child’s consumption. Furthermore, it comes at a time when the beneficiary no longer is a child and, therefore, no longer subject to credit rationing. The purpose of a bequest cannot then be that of relaxing a temporary liquidity constraint, but only that of altering the distribution of wealth between the parent and the child.} If (21) is binding for some $t$, the \textit{laissez faire} is inefficient.

3.2 The family

The assumption that individuals interact only through the market is clearly unrealistic. In the real world, individuals interact also through organizations such as families, clubs, and interest groups. In particular, decisions regarding fertility and the intergenerational allocation of resources tend to be coordinated by families.\footnote{In a sense, the Becker-Barro model is a model of the family. But it a very particular sort of family, where only one person, the founder, takes all the decisions.} In game-theoretical language, any such type of organization is a \textit{coalition}, a subset of the population whose members are better off re-distributing their endowments among themselves, than through the grand coalition that we call the market.

3.2.1 A family constitution

A useful way of characterizing an organization is to describe its fundamental rules, its \textit{constitution}. Economic theory tells us that there may be circumstances in which it is in everyone’s interest to agree first on a constitution, allowing agents to safely renounce the dominant strategy in a prisoner’s dilemma type of situation, and then optimize subject to that constitution (Buchanan, 1987). Cigno (1993) puts forward the idea of a ”family constitution”, and establishes conditions under which this is self-enforcing in the sense that it will be in the best interest of each family member to obey it, and to have it obeyed. Cigno (2000) identifies circumstances in which a constitution is self-enforcing also in the stronger sense that, once established, it will never be amended. Empirical testing cannot reject the hypothesis that behaviour is constrained by such constitutions.\footnote{See, for example, Cigno, Giannelli and Rosati (1998).}

At any given date, a family consists of a number of individuals at different points of the life-cycle. Age differences are important, because they provide an opportunity for mutually beneficial exchanges between members of the same family.\footnote{Such opportunities may arise also from differences of sex or other personal char-} A family constitution is defined as a set
of (unwritten, typically unspoken) rules prescribing, for each date $t$, the minimum amount of income, $z^t$, that each adult must transfer to each of her children (if she has any), and the minimum amount of income, $x^t$, that she must transfer to her parent. Such transfers are subject to the pro viiso that nothing is due to a parent who did not herself obey the rules; this makes it in every adult’s interest to punish transgressors. That is important, because only an adult can punish another adult; neither children nor old people have the means to do so. Supposing, for simplicity, that $r$ (hence $k$ and $y$) is constant over time, it seems natural to postulate $x^t = x$, $z^t = z$ for all $t$.\footnote{This avoids some awkward analytical problems}

Let us assume that people are self-interested, so that the lifetime utility of each person is given by (4). If members of different generations cooperate in such unpromising circumstances, all the more they will if they harbour altruistic sentiments. The existence of a family constitution confronts each adult with a choice of two strategies: comply with the constitution (cooperate), or go it alone in the market (defect). Since children cost their parents something (at least $p$), but will only bring a return if the constitution is complied with, it is clear that go-it-aloners will have no children. For reasons that will become clear in a moment, compliers have no interest in lending to the capital market, and are not allowed to borrow more than a certain quota (normalized to zero). Being self-interested, compliers do not transfer more than the minimum required by the constitution.

Denoting the amount lent to the market by $s$, the pay-off to going it alone is

$$v(r, w) = \max_s u_1 (w - s) + u_2 (rs).$$

(24)

For any given $(r, w)$, the choice of $s$ satisfies

$$\frac{u_1'}{u_2'} = r.$$  

(25)

The effects of changes in $r$ or $w$ on the pay-off of this strategy are

$$v_w = u_1' (w - s), v_r = su_2' (rs).$$

(26)

The pay-off to complying, provided that the agent’s children also comply, is

$$v^*(x, w, z) = \max_n u_1 (y - x - (p + z) n) + u_2 (xn).$$

(27)

acteristics, but we have assumed them away because we are interested primarily in exchanges between generations.
For any given \((x, w, z)\), the choice of \(n\) satisfies

\[
\frac{u'_1}{u'_2} = \frac{x}{p + z}.
\] (28)

The effects of changes in \(x, y\) or \(z\) on the pay-off of this strategy are

\[
v^*_x = -u'_1 (w - x - (p + z) n) + nu'_2 (xn),
\] (29)

\[
v^*_w = u'_1 (w - x - (p + z) n),
\] (30)

\[
v^*_z = -nu'_1 (w - x - (p + z) n).
\] (31)

If

\[
v^*(x, w, z) \geq v(r, w),
\] (32)

complying is the best response to everyone else doing the same. The set of ”comply” strategies (one for each member of each generation) is thus a Nash equilibrium. Since complying implies threatening one’s own parent of punishment if she does not also comply, and since the threat is credible, because carrying it out is clearly in the interest of the person making the threat, the equilibrium is sub-game perfect.\(^{17}\)

For a complier, having a child is a form of investment, costing \(p + z\) in the current period, and yielding (if the child in turn complies) \(x\) in the next. Since a complier must pay a fixed amount \(x\) to her parent, irrespective of how many children she has, a necessary condition for (32) to be true is that the return to having a child is strictly larger than the return to saving,

\[
\frac{x}{p + z} > r.
\] (33)

Were that not so, there is in fact no way that an agent could recover the fixed cost of complying.

If (33) holds, there can be only two reasons for a complier to lend to the market. One has to do with the existence of a physiological ceiling on fertility. If that is binding, the agent cannot have as many children (cannot buy as many entitlements to future transfers) as she would like, and may thus find it optimal to top-up her family entitlements with conventional assets; in other words, she may save (Cigno, 2000). The other is uncertainty. If domestic entitlements are vulnerable to unpredictable events, such as the premature death of a child,\(^{18}\) a risk-averse complier

\(^{17}\)In equilibrium, the threat is never carried out because everybody complies.

\(^{18}\)Other examples of such an event are a child of insufficient earning ability, or changes in the economic environment (e.g., in government policy, or in the internationally determined interest rate) such that the existing constitution is no longer self-enforcing.
may well find it optimal to do some precautionary saving (Rosati, 1996). Allowing for either of these possibilities complicates the exposition without altering the results in any relevant way.

What about borrowing? While making it disadvantageous for compliers to lend to the market, (33) makes it advantageous for them to borrow from the market in order to finance additional births. But there are limits to this arbitrage operation. One is that fertility will eventually hit its physiological ceiling. The other is that there is typically no legal mechanism through which entitlements arising from an informal family arrangement can be transferred to another person. If it cannot be transferred, an entitlement cannot be offered as collateral to obtain credit from the market.\(^{19}\)

The set of constitutions that can be supported by a sub-game perfect Nash equilibrium consists of all the \((x, z)\) pairs which satisfy (32). Since children have no income, \(z\) cannot be negative. Since agents are adults, they would happily subscribe to a constitution that does not oblige them to make transfers to their children (in addition to the subsistence minimum included in \(p\)) in the current period, but they would not countenance a constitution that does not entitle them to receive transfers from their children in the next period. Therefore, all points of the set will satisfy \(x > 0, z \geq 0\).

The boundary of this set is the locus of the \((x, z)\) pairs that make (32) into an equation. Its slope is

\[
\frac{dz}{dx} = \frac{(p + z)n - x}{nx}, \tag{34}
\]

Since

\[
\frac{d^2z}{d(x)^2} = -\frac{p + z}{(x)^2} < 0, \tag{35}
\]

\(z\) is at a maximum where

\[
\frac{x}{p + z} = n. \tag{36}
\]

These properties are illustrated in Figure 1.

It follows from (26) and (29) – (31) that a rise in \(w\) makes the set of constitutions which can be supported by a sub-game perfect Nash equilibrium larger (the boundary shifts outwards). Intuitively, that is because, in view of (33), the marginal utility of current income is greater

\(^{19}\)We are assuming that compliers are not allowed to borrow from the market at all, but nothing of substance changes if we allow borrowing up to some positive maximum, smaller than \(nz\).
for a complier, than for a go-it-aloner. Conversely, a rise in \( r \) makes the set smaller (the boundary shifts inwards). The intuition, here, is that the minimum rate of return to children needed to make complying at least as attractive as going it alone in the market gets larger as the interest rate rises. If \( w \) is sufficiently low, or \( r \) sufficiently high, the set of potentially self-enforcing constitutions is empty.\footnote{Under our assumption that all individuals are the same, there either is, or is not, a self-enforcing constitution for all families. Allowing for heterogeneity, however, some families will have a self-enforcing constitution, and others will not (or rather, if we identify a family with its constitution, some individuals will not have a family). Changes in the interest rate will then shift the margin between compliers and go-it-aloners (Cigno, 2000). Other things being equal, a rise in the interest rate, or easier access to financial markets by wider strata of society, should thus result in a lower rate of compliance with family constitutions. Evidence of that is found by Cigno and Rosati (1992) in a developed country, by Foster and Rosenzweig (2000) in developing ones.}

### 3.2.2 Selecting a constitution

Given that an infinite number of \((x, z)\) pairs may satisfy (32), and an infinite number of constitutions could thus be sustained by a sub-game perfect Nash equilibrium, which will prevail? Cigno (1993) suggests that the family founder will choose the constitution which suits her best. Since the founder is a selfish adult, she will obviously favour the one that prescribes the largest enforceable\footnote{In the limited sense that the constitution prescribing it is a sub-game perfect Nash equilibrium.} transfer to the old, and no transfers (above the subsistence level) to children. In Figure 1, this is represented by point \((0, x_0)\).\footnote{If the associated \( n \) violates the physiological ceiling on fertility, the founder will choose the constitution with the \( n \) that just meets that constraint.}

Cigno (2000) offers an alternative selection criterion, akin to the renegotiation-proofness concept developed by Bernheim and Ray (1989), and Farrell and Maskin (1989).\footnote{But these contributions refer to a situation where the players are always the same, not to an overlapping generation model like Cigno (1993, 2000), where the players change at each round.} At any date \( t \), any of the current adults is at liberty to propose a new constitution. Will subsequent generations take any notice? Not if (i) the old constitution satisfies (32), and (ii) no other constitution satisfying (32) makes generations \( t, t + 1, t + 2, \ldots \) better-off. In other words, a constitution is renegotiation proof if, in addition to being a sub-game perfect Nash equilibrium, it allocates consumption across generations in a Pareto-optimal way. The argument runs as follows.

If there are no free lunches to be dished out, the only way a member of generation \( t \) could offer generation \((t + 1)\) a better deal than the...
existing constitution, and not loose in the bargain, is by paying generation \((t - 1)\) less than the existing constitution requires. That, however, would mean defaulting on the existing constitution. All members of generation \((t + 1)\) would then be better off upholding the existing constitution, which entitles them to pay nothing to generation \(t\), than acquiescing to the proposed new one.\(^{24}\) Once established, a constitution satisfying the double requirement of being a sub-game perfect Nash equilibrium, and an intergenerational Pareto optimum is thus unamendable.\(^{25}\) We now adapt a procedure developed by Anderberg and Balestrino (2002) for deriving the properties of the winning constitution in a context similar to the present one (see sub-section 4.2).

For any given \(n\) and \(w\), a constitution prescribing \((z, x)\) is a Pareto optimum if it maximizes

\[
u (n, z, x) \equiv u_0 (z) + u_1 (w - x - (p + z) n) + u_2 (x n),
\]

and thus satisfies

\[
\frac{u'_0}{u'_1} = n = \frac{u'_1}{u'_2}.
\]

Given such a constitution, parents would choose to have the number of children which satisfies

\[
\frac{u'_1}{u'_2} = \frac{x}{p + z}.
\]

Does any of the constitutions that can be sustained by a sub-game perfect Nash equilibrium satisfy (38) – (39)? The one favoured by the family founder, \((0, x_0)\), clearly does not.\(^{26}\) The one which maximizes \(z\) satisfies (36), as well as (28). Of all sustainable constitutions, the one

\(^{24}\)Anderberg and Balestrino (2002) point out that this corresponds to the weak notion of renegotiation-proofness (internal consistency). The strong notion (external consistency) requires an equilibrium to be undominated by any weakly renegotiation-proof equilibrium.

\(^{25}\)Browning (1975), referred to in Section 6 below, makes the point that efficiency cannot be achieved in a democracy because children have no veto power. Does the finding that a renegotiation proof family constitution allocates consumption efficiently then imply that children have veto power on family decisions? It does not. A renegotiation proof family constitution is efficient because any inefficient one is amendable by current adults. We shall see that the same mechanism is difficult to reproduce at societal level.

\(^{26}\)That, incidentally, is the one that would be legislated in a direct democracy (Browning, 1975).
which prescribes the largest transfer to each child is then renegotiation proof.\textsuperscript{27}

This proposition is illustrated in Figure 2. The lifetime utility of the representative family member is maximized at the point, \((z^*, x^*)\), where the marginal rate of substitution of \(c_0\) for \(c_1\) is equal to the marginal rate of substitution of \(c_1\) for \(c_2\), and the common value of these marginal rates is equal to the number of children. The curves circling around that point are the contours of (37). Away from the maximum, the slope of the contour,

\[
\frac{dx}{dz} = \frac{u_0' u_1' - n}{u_2' u_1' - n},
\]

may be positive or negative. At any point in the set of Nash equilibria, \(\frac{u_1'}{u_2'}\) is equated to \(\frac{x}{p+z}\) by choice of \(n\). At point \((z^*, x^*)\) of that set, where the amount transferred to each child is the largest possible, the chosen \(n\) is equal to \(\frac{x}{p+z}\) in view of (36). The constitution that prescribes the largest \(z\) is renegotiation proof.\textsuperscript{28}

We have thus found that the winning constitution allocates consumption efficiently across generations, and over the life-cycle of each generation. In view of (33), however, it induces parents to choose \(n\) greater than \(r\). Recalling that the socially optimal \(n\) is no greater than \(r\), this then means that fertility will be too high. Therefore, the existence of spontaneous intra-family arrangements does not guarantee a social optimum: parents will have too many children. The reason is insufficient commitment. Selfish agents will subscribe to a system of voluntary transfers only if it offers a return to investing in children which is higher than the return to investing in conventional assets.

The upward distortion of the fertility rate may be mitigated or even reversed at the aggregate level if the population is heterogeneous. If members of the same generation differ in their ability to produce income, or in the cost of producing children, one can envisage a situation

\textsuperscript{27}Notice that the utility of the representative family member was maximized subject only to the restriction that life-cycle re-allocations are to be achieved by means of intra-family transfers. It just so happens that the solution is an element, the one with the largest \(z\), of the set of sub-game perfect Nash equilibria (for an example, see the next footnote). Were that not so, we would have had to re-optimize subject to (32).

\textsuperscript{28}For example, if \(U = \ln (c_0) + \ln (c_1) + \ln (c_2)\), the renegotiation-proof constitution prescribes \((z = \frac{w-pn}{3n} + x = \frac{w-pn}{n})\).
where some agents are governed by self-enforcing constitutions, and others are not. That being the case, some agents ("compliers") would have children, others ("go-it-aloners") would not. Cigno (2000) determines endogenously the proportion of agents who will comply with some family constitution, and that of agents who will go it alone in the market.

### 3.2.3 Altruism and personal services

A number of authors, including Bernheim, Schleifer and Summers (1984), Cox (1987), and Cox and Jakubson (1995), allows for the possibility that money is not a perfect substitute for the personal services ("attention") of the agent’s own children. To allow for that, and for the possibility that parental services to children also have no perfect market substitute, Cigno and Rosati (2000) re-formulate the family constitution in such a way that it requires each adult to transfer a certain level of utility, rather than income, to her parent and children. As these services are available only to compliers, and money can be substituted for them only at a diminishing marginal rate, extending the model in this way raises the marginal benefit of complying. By allowing the giver to choose the cost-minimizing combination of income and own time with which to satisfy the constitutional requirement, it also reduces the marginal cost of complying. Introducing personal services without a perfect market substitute has thus the effect of relaxing the conditions for the existence of a self-enforcing family constitution (by making it more likely that a family scheme can offer a higher return than the capital market).\(^{29}\)

Would it make a difference if people cared for the well-being of their progeny as in Becker and Barro (1988)? As far as we are aware, the case has not been worked out in the literature, but it is not difficult to see what would happen. If the utility function is (14), not only compliers, but also go-it-aloners will have children. Since go-it-aloners get only direct utility from having children, while compliers get also indirect utility (via transfers), however, it remains true that the latter will have more children than the former as in the basic model. Since (33) must still hold, it also remains true that compliers will have more children than is socially desirable. The reason is again insufficient commitment. Adults would be willing to support their children even if they got nothing in return. Given convex preferences, however, they would not support each

\(^{29}\)It also provides a rationale for the imbalance between monetary flows to the young and to the old observed in developed economies. Having relatively large pensions or accumulated savings on average, the old attach higher marginal utility to personal services without perfect market substitutes, than to income transfers. It is thus cheaper for their grown-up children to provide them with any given level of utility by giving them personal attention, than by giving them money (Cigno and Rosati, 2000).
child at a sufficiently high level. Furthermore, they would not support their aging parents.

3.3 The government

We have seen that the *laissez faire* solution is generally different from the social optimum. We now look at ways in which this can be remedied by policy under the assumption that (i) parental actions are visible, and (ii) the government does not need to account for its policies to the electorate. The problem of hidden parental actions is dealt with in Section 5,\(^{30}\) that of political acceptability in Section 6.

Let us start by supposing that there are no self-enforcing family constitutions, and that the market is thus the only spontaneous coordination mechanism available. Groezen, Leers and Meijdam (2002) show that a Millian optimum can be implemented by the introduction, side by side, of a pay-as-you-go pension scheme, and a system of child benefits financed by a lump-sum tax on adults. Analogous results were obtained in earlier studies by Peters (1995), and Kolmar (1997). Groezen *et al.* develop the analysis under the assumption that people derive utility from their own consumption, and from the number (not the consumption, or the lifetime utility) of children. They also use a specific functional form for \(u_i(\cdot)\), and assume zero social time preference \((\delta = 1)\). But the argument has more general validity. We examine the case where people derive utility from their own lifetime consumption only, so the utility function is (4) as in our basic model.

Let \(\eta\) be a lump-sum benefit payable to each old person, and \(\theta\) a lump-sum contribution payable by each adult. If

\[
\eta = \theta r, \tag{41}
\]

the scheme does not force an intergenerational transfer. Similarly, let \(\varphi\) be the child benefit rate, payable to adults for each child they have, and \(\tau\) a lump-sum tax, payable by each adult. If

\[
\varphi n = \tau, \tag{42}
\]

there is again no forced intergenerational transfer, but income is redistributed from adults without, to adults with children. Having assumed that parental actions are observable, the government can make sure that parents pass the subsidy \(\varphi\) on to their children, or pay the subsidy directly to the child (*e.g.*, in the form of free and compulsory

\(^{30}\)If all members of the same generation are the same, hidden characteristics are hardly an issue.
education, free school meals, etc.) Were that not true, we would have an agency problem (see next section).

Let \( c_i^* \) denote the socially optimal steady-state value of \( c_i \). Let the credit ration faced by children be equal to zero. For any given \( r \), the government can implement a Millian social optimum by setting \( \eta = c_2^* \), \( \theta = \frac{\tilde{\varphi}}{r} \), and offering taxpayers the following "forcing contract":\(^{31}\)

\[
\varphi = p + c_0^* \quad \text{and} \quad \tau = (p + c_0^*) \delta r \quad \text{if} \quad n = \delta r \quad \text{and} \quad c_0 = c_0^*,
\]
\[
\varphi = 0 \quad \text{and} \quad \tau = \tau' > (p + c_0^*) \delta r \quad \text{otherwise}.
\]

(43)

An agent will then consider two alternative courses of action: either have \( r \) children, spend \( p + c_0^* \) for each child, and save nothing; or have no children, and save some positive amount, \( s \). Given (4), the first of these options gives utility

\[
u_1 (w - \theta) + u_2 (\eta).
\]

The second yields

\[
\max_s u_1 (w - \theta - \tau' - s) + u_2 (\eta + sr).
\]

(45)

By setting \( \tau' \) sufficiently large, the government can induce each agent to choose the first of these two courses of action. Consequently, \( c_i = c_i^* \) \((i = 0, 1, 2)\), and \( n = \delta r \). In view of (9) and (41), pensions can be financed on a pay-as-you-go basis if social time preference is absent \((\delta = 1)\), so that \( n = r \) at the optimum, but not otherwise.

Therefore, a Millian social optimum can be implemented by the combination of a public pension system and a child benefit scheme (bolstered by a penalty for parents who do not have the socially optimal number of children, or spend less than is socially optimal for each child). This system of public transfers looks remarkably like a family constitution, with \( \eta \) in the place of \( x \), and \( \varphi \) in that of \( z \), but with an important difference. Unlike the family, the government has the power to coerce. It is thanks to this power,\(^{32}\) and to the assumptions that the government \((i)\) is free to choose its policies without worrying about re-election, and \((ii)\) can observe, an thus control, individual actions, that a first best can be achieved.

Under the assumption that all members of the same generation are the same, self-enforcing family constitutions make the government’s task

\(^{31}\)The expression comes from the principal-agent literature. It is applicable in any situation like the present one, where the agent’s action is observable by the principal.

\(^{32}\)This power makes it unnecessary for the government to pay over the odds in order to induce agents to participate in its pension scheme.
easier. Since each family will then have the same constitution, all the
government has to do is get parents to have the right number of children
by offering them a forcing contract. If agents are heterogeneous, however,
some will be governed by a self-enforcing constitution, others will not.
Then, some will have children, others will not (or, if they are dynastically
minded, will have fewer). As already pointed out, the aggregate fertility
rate could then be "right" by chance. If it is not, heterogeneity will
make the government’s task harder, rather than easier, because it makes
it more costly for it to find out all that there is to be known about each
individual. A first best may then be out of reach, but a public pension
system and a child benefit scheme would still be central to a second-best
policy.\textsuperscript{33}

4 Education

Reflecting the growing interest in human capital as the mainspring of
economic growth, a series of papers on intergenerational transfers, in-
cluding Cremer, Kessler and Pestieau (1992), Docquier and Michel (1999),
Kaganovich and Zilcha (1999), Pecchenino and Utendorf (1999), Kem-
nitz (2000), Anderberg and Balestrino (2002), Boldrin and Alonso (2002),
focuses on the efficiency of private provision for children’s education,
rather than consumption. Since education is a factor in the production
of human capital, and human capital is in turn a factor, along with
ordinary capital, in the production of income, this poses a problem of
portfolio choice. To compensate for this complication, the literature in
question makes a number of simplifications. The first is to assume that
fertility is exogenous. The second is that education occurs only in pe-
riod 0, and saving only in period 1, of a person’s life. There is thus no
problem of timing of asset formation. Third, assets last only one period:
human capital vanishes at the end of period 1, capital self-destroys at
the end of period 2.\textsuperscript{34}

With the exception of Pecchenino and Utendorf, this literature as-
sumes that agents are moved by sheer self-interest as in our basic model.
Except for Cremer \textit{et al.}, who introduce filial attention in the utility
function of parents, all these authors assume that utility depends on the
consumption of market goods only. Further assuming that consumption
in period 0 is a constant (normalized to zero), and that the only effect
\textsuperscript{33}Cigno and Rosati (1992, 1996, 1997), and Cigno, Casolaro and Rosati (2003)
find evidence that a public pension system without intergenerational debt reduces
the aggregate fertility rate, and generally raises the aggregate household saving rate.
Child-related benefits raise the aggregate fertility rate, but may discourage household
saving.

\textsuperscript{34}Strictly speaking, therefore, there is no asset \textit{accumulation}. 
of education is to increase future earning capacity (education *per se* has no utility), the life objective of each member of generation $t$ may then be written as

$$U^t = u_1 \left( c_1^t \right) + u_2 \left( c_2^t \right).$$

(46)

The income produced by an adult at date $t$ is given by

$$y^t = f \left( h^t, k^t \right),$$

(47)

where $f \left( . \right)$ is a constant-returns-to-scale production function with the usual properties, $h^t$ the stock of human capital, and $k^t$ the stock of capital, all in per-adult terms. The stock of human capital is determined by

$$h^t = g \left( e^t, \xi \right),$$

(48)

where $g \left( . \right)$ is another production function, with properties analogous to those of $f \left( . \right)$, $e^t$ is the education that each member of generation $t$ received at date $t - 1$, and $\xi$ is the endowment of human capital that a person receives at birth (”native talent”).

Continuing to assume a small open economy and perfect capital mobility, the interest rate $(r - 1)$ is still exogenously determined, but this does not pin down the capital/labour ratio as in the one-asset case, because capital is now substitutable with human capital. Therefore, in general, $k^t$ and $h^t$ (hence factor prices) may vary over time. Given constant returns to scale, however, the asset mix, and thus the price of $h^t$, is determined by $r^t$. The resource constraint is now

$$f \left( k^t, g \left( e^t \right) \right) - r^t d^t = \frac{c_2^{t-1}}{n^{t-1}} + c_1^t + \left( e^{t+1} + k^{t+1} - d^{t+1} \right) n^t.$$  

(49)

The first-order conditions for the maximization of social welfare, subject to (49), include (6) and the second of the equations in (7). Additionally, there is now a portfolio condition,

$$f_h \left( h^t, k^t \right) g_e \left( e^t, \xi \right) = r^t = f_k \left( h^t, k^t \right),$$  

(50)

stating that the return to education must equal the return to investing in capital. As fertility is exogenous, the marginal conditions for a (first-best) social optimum coincide with those for a Pareto optimum.

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35With exogenous fertility, it does not matter whether the social welfare function is of the Benthamite or of the Millian variety. Since the only difference between the two is that the time preference factor is $\delta n$ in the former, $\delta$ in the latter, using one or the other makes no qualitative difference when $n$ is not a choice variable.

36The first equation has disappeared because there is no consumption in period 0.
4.1 Market equilibrium and government intervention

The questions posed by Cremer et al. are analogous to those raised in our sections 2 and 3. Assuming that the young are rationed in the capital market, what is the socially optimal level of transfers from parents to young children, and from grown-up children to elderly parents? Is laissez faire efficient? A difference is that transfers to children now buy education, rather than consumption. Another is that fertility is exogenous (population grows at an exogenously given rate, conveniently assumed equal to the interest rate, so that a pay-as-you-go pension system can be efficient).37 There is no family constitution prescribing parental support for children, but the authors postulate, like Bernheim et al., that parents can use the promise of a bequest to extract a maximum of attention from their children.38 The question is whether investing in their children’s education at the efficient level will raise the attention received sufficiently to make it in their own interest to carry out the investment in the absence of policy. The answer is no. Under certain assumptions on the form of the utility function, an efficient solution can be implemented by the combination of a public education system with a public pension scheme.

Much the same questions are posed by Boldrin and Alonso, but personal services are left out of the picture, and children are assumed to take their own lifetime decisions as in Modigliani. All that is different here, compared with the model of our sub-section 3.1, is that people choose how much to spend for their own education, rather than how much to spend for their consumption in period 0 (but this is enough to change the efficiency conditions). Let us look at this model in a little more detail.

In the absence of credit rationing, a person born at date \( t - 1 \) chooses

37 But there is obviously no justification for assuming that two exogenously given constants, the rate of population growth and the rate of interest, will be equal.

38 The idea is that parents pre-commit to leaving their entire estate to the child who gives them the greatest attention. Cigno (1991) argues that children can easily counter this strategy by making a (perfectly legal) contract among themselves, whereby they agree that (a) one of them will give the parent a minimum of attention, while the others will give none, and (b) the estate will be re-distributed, after the parent’s death, so as to give each one of them the same share (or slightly more to the one who did something for the parent). If successful, the strategy proposed by Bernheim et al. would allow the parent to extract the entire surplus from the children. The one suggested by Cigno would allow the children to extract the entire surplus from the parent.
\((e^t, c_1^t, c_2^t)\) to maximize (46), subject to
\[c_1^t = h^t w^t - e^t r^t - s^t,\]

where \(w^t\) is now defined as the price of human capital at date \(t\), rather than the wage rate (now given by \(h^t w^t\)) as in previous sections, and
\[c_2^t = s^t r^t.\]

Given (48), in period 0, this person will borrow from the capital market in order to finance her education to the point where the marginal benefit equals the marginal cost,
\[w^t g_e (e^t, \xi) = r^t.\]

In period 1, the same person will save (lend to the capital market) to the point where her marginal valuation of her own current consumption equals the marginal cost. That yields the familiar condition (6).

Assuming perfect competition, employers equate the marginal product of capital to the interest factor,
\[f_k (h^t, k^t) = r^t,\]

and the marginal product of human capital to the price of the same,
\[f_h (h^t, k^t) = w^t.\]

Therefore, both the portfolio and the life-cycle conditions are satisfied.

In the absence of binding credit rations, individual decisions would then allocate consumption efficiently over the life-cycle of each generation. Using standard arguments, Boldrin and Alonso show that individual decisions allocate consumption efficiently also across generations.\(^{39}\) The difference between this case and the one examined in Cremer et al. is quite clear. There, the education of each generation is decided upon, and paid for, by the previous generation. Inefficiency may then arise from a coordination failure between adjoining generations. Here, by contrast, everyone looks after himself from the word go. In the absence of externalities or public goods, there can then be inefficiency only if children are rationed in the credit market. If the ration is binding, the economy will invest too little in human capital.

The remedy proposed by Boldrin and Alonso is analogous to that offered by Groezen et al. for the case in which transfers to young children

\(^{39}\) Compared with the one-asset case, additional assumptions are needed to ensure the existence of a dynamic equilibrium in the two-asset case.
buy consumption, rather than education. They envisage two lump-sum taxes, both levied on adults. One is used to pay for pensions (transfers to the old), the other to pay for education (transfers to children). The only difference between this, and the scheme described in sub-section 3.3 is that, as fertility is now exogenous, there is no need for a forcing contract to get parents to have the socially desirable number of children. If the pension is set equal to the optimal $c_2$, and the educational grant equal to the optimal $e$, individual choice will yield the social optimum. If parents altruistically provide education for their children as in Pecchenino and Utendorf, however, the introduction of a public pension system may crowd out voluntary provision for education, and reduce welfare. We will see that the same is true if private transfers are governed by family constitutions.

4.2 Family constitutions once again

Anderberg and Balestrino (2002) pose the same questions as Cigno (1993, 2000), but in an education context. Are there circumstances in which a family constitution (they call it a "family norm") involving support for children and for the old would be self-enforcing? And, given that such circumstances apply, will the constitution bring about an efficient allocation of consumption? In this model too, an (adult) agent can choose to either comply with the constitution, or go it alone in the market. Again, the constitution prescribes the amount $x$ that each adult must pay to her elderly parent, and the amount $z$ that she must pay to each of her children (in addition to bearing the unavoidable cost, $p$). The main difference between this and the model of sub-section 3.2 is that $z$ now pays for education, rather than consumption, and that $n$ is exogenous. Additionally, adults can now choose how to allocate their time between work and leisure (otherwise, with the number of children given, a complier would have nothing left to decide).\footnote{Strictly speaking, a complier can also choose how much to save. However, as we saw in sub-section 3.2, saving may be different from zero only if the pay-off to complying in uncertain as in Rosati (1996). Anderberg and Balestrino (2002) allow for this possibility as an extension to their basic model.} Lifetime utility is given by (46), but $c^*_t$ is defined net of the income-equivalent of the disutility of labour.\footnote{Thus modelled, the time allocation decision plays a secondary role in the architecture of the model.}

The first step is again to define the set of the constitutions that can be supported by a sub-game perfect Nash equilibrium. The second is to select a constitution that is also renegotiation-proof. A constitution belonging to the set of Nash equilibria is characterized by an implicit
return factor to the total amount of money that a complier pays into the family scheme, \( \frac{nx}{(p+z)n+z} \), strictly higher than the market interest factor, \( r \). As the largest possible value of this factor (if \( p + z = 0 \)) is \( n \), the existence of a Nash equilibrium implies that \( n \) is greater than \( r \).

Superficially, that is the same as the result of sub-section 3.2, but the reasoning is quite different. What has to be greater than \( r \), there, is the return to having a child (given that the decision to comply has been taken), \( \frac{x}{p+z} \), equal to \( n \) if the constitution is renegotiation proof. Therefore, the winning constitution is so designed that agents choose \( n \) greater than \( r \). Here, by contrast, fertility is exogenous. Therefore, the return to having an extra child is irrelevant, and \( n \) will be smaller than \( r \) only by chance. If it is not, the constitution will be inefficient.

The selection criterion is again that the constitution must not only be a sub-game perfect Nash equilibrium, but also renegotiation proof. It must then maximize the lifetime utility of the representative agent, subject to the constraint that it is an element of the set of sub-game perfect Nash equilibria. In the model examined in sub-section 3.2, the constraint is never binding. Here, by contrast, it may. If it is, the family will underinvest in the education of its own children. Therefore, in the Anderberg-Balestrino model, the winning constitution need not allocate consumption efficiently.

Why is there such a difference of results between two apparently very similar models? The answer lies in the very concept of efficiency. As Pareto comparisons presuppose a given number of individuals, an allocation of consumption over a certain population profile is efficient if it is not dominated by any other allocation relating to the same profile. In the model of sub-section 3.2, agents choose fertility taking the constitution as given. The constitution can then be designed in such a way, that it generates an efficient allocation of consumption over the population profile implied by the associated fertility rate. Here, by contrast, there may then be no self-enforcing constitution that allocates consumption efficiently, because fertility is exogenously determined.

An education policy may thus be justifiable even if self-enforcing family constitutions are in place. If that is the case, the government has a choice of two possible courses of action. One is to introduce a pension scheme and a system of educational grants side by side as suggested by Boldrin and Alonso. This would replace voluntary transfers through the family network with compulsory transfers through the tax-benefit system. The alternative is to take care not to crowd out voluntary transfers, and change the economic environment through the fiscal

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instrument in such a way that self-enforcing family constitutions will prescribe higher educational investments in children.

If parental actions are observable, as we have assumed so far, the first-best policy will again take the form of a forcing contract, this time on human capital investment, rather than on number of children as in sub-section 3.3. If they are not, the first best is generally out of reach, Anderberg and Balestrino derive the properties of the second-best policy in the context of their own model, under the assumption that only anonimous market transactions are observable and, therefore, that only distortionary policies are available. If the actions that the government cannot observe are those which most directly affect the welfare of children, there is also an agency problem. We turn to that in the next section.

5 Agency problems

Suppose that the child’s utility depends in some way on actions taken by their parents, and that some of these actions are not observable by the government. It could be, for example, that the allocation of resources within the household is not observable from outside. Public transfers intended for the benefit of children could then end up as parental consumption. Within the context of the model of Section 4, it is natural to assume that a child’s scholastic success depends not only on observable factors, such as school attendance, or specifically educational expenditure, but also on the allocation of general household expenditure (or parental attention), which cannot be so easily monitored by the government.

If the child’s educational achievement were fully determined by parental actions, the government could infer the level of the unobservable action from the observation of the outcome. That is not possible, however, if the outcome depends also on a random factor (“luck”). If that is the case, the government cannot use a forcing contract to get parents to choose the right level of the action. We have then an agency problem, with the government in the role of principal, and parents in that of agents.44

In Cigno, Luporini and Pettini (2002), parents choose the level of the action that affects their children’s utility under conditions of uncertainty about the effect of the action, taking government policy as given. The government chooses its policy so as to provide parents with the incentive to take the desirable level of the action. The authors use this model to

43 But Anderberg and Balestrino make no such assumption.
44 If children figure in the objective function of the government, any optimal taxation problem is a principal-agent problem in the broad sense. It is a principal-agent problem in the technical sense if actions can be neither observed nor inferred.
examine the possible advantages of making child benefits conditional on number of children, and on the children’s performance. We re-expose the model using the notation of Section 4.

5.1 Parents as agents

Taking the tax system as given, the stock of human capital, $h^{t+1}$, can be measured by the value, discounted to time $t$, of the taxes that a person born (and educated) at $t$ pays at $t+1$. As in all the models considered so far, parents (agents) derive utility from their own consumption, now denoted simply by $c$ (adulthood and old age are collapsed into one period). Possibly, they derive utility also from the number of children that they have, $n$, and from the welfare of each of these children, proxied by $h$. The latter is determined by (48) as in Section 4, but $\xi$ is now a random variable (we may continue to interpret this as natural talent, or think of it, more generally, as luck) with given density. Then $h$ itself is a random variable with density $\phi(h, e)$, derived from that of $\xi$ using (48).

The agent’s expected utility is given by

$$E(U) = \int U[c + v(h)n] \phi(h, e) dh,$$

where $U(.)$ is the ex-post utility function, assumed concave to indicate risk aversion. We may interpret $v(h)$ as the consumption-equivalent of the pleasure that altruistic parents derive from their children’s well-being (proxied by $h$). If agents are selfish as in our basic model, $v(h) \equiv 0$. The household budget constraint is then

$$c = y + [\varphi(h) - w(e) - p]n,$$

where $y$ is the parent’s income, $\varphi$ the child benefit rate (possibly conditional on $h$), $p$ the unavoidable part (independent of $e$) of the cost of having a child, and $w(e)$ the per-child cost of the action $e$. If $e$ is time, $w(e)$ includes an opportunity cost, and $y$ is then to be interpreted as full income. Alternatively, we may assume that parents who invest in their children’s education during the current period are entitled to receive transfers from them during the next period, and interpret $v(h)$

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45Given the tax system, tax payments may be taken to be a monotone function of earnings, and thus as good a measure of a person’s human capital as any other.

46To ensure an interior solution to the agent’s optimization problem, $w(.)$ is assumed increasing and convex (increasing marginal cost of $e$). This is justified by noting that the cost of the action includes the opportunity-cost of household resources in fixed supply.

47In other words, that a self-enforcing family constitution like those discussed in earlier sections is in operation.
as the present value of these transfers. Supposing, as we did in earlier sections, that it is not possible to borrow from the market against the informal promise of a transfer from a relative, and further assuming that it is not possible to insure against (one’s own children’s) bad luck, the budget constraint is still (57).

If fertility is exogenous, parents choose $e$ to maximize (56), where $c$ is given by (57). The first-order condition,

$$-w'(e)n \int U'(h, e)dh + \int U\phi(h, e)dh = 0,$$

(58)
tells us that parents equalize the expected marginal cost to the expected marginal benefit of $e$.

If fertility is endogenous (and perfectly controllable by parents, as we have assumed so far), parents choose also $n$. There is then the additional first-order condition,

$$\int [\varphi(h) + v(h) - w(a) - p] U'(h, e)dx = 0,$$

(59)
that parents must procreate to the point where the cost of an additional child equals the expected benefit. Notice that, if $v(h) \equiv 0$, the benefit of having children and expending resources on them can come only from $\varphi$. Therefore, if parents are not altruistic, $e$ can be positive only if $\varphi$ increases with $h$. With endogenous fertility, children will be born only if $\varphi$ is positive, and at least as large as $p$.

5.2 The government as principal

Assuming that the number of agents and, thus, the number of future taxpayers is ”large”, so that the government does not face any revenue uncertainty, we may write the government budget constraint in terms of expected tax revenue,

$$n \int [h - \varphi(h)]\phi(h, e)dh \geq 0.$$

(60)

Given Benthamite social preferences, the principal aims to maximize (56), subject to (60) and to the appropriate incentive-compatibility constraints.

Comparing (60) with (57) makes it clear that, if $\varphi$ did not depend on $h$, a child would always count more for society than for the representative parent. That is because the (atomistic) parent has no way and no reason to take into account the effect of her choice of $e$ and, if fertility is endogenous, $n$ on the government budget constraint. The presence of this fiscal externality is enough to justify public intervention. An
additional justification is in the absence of perfect insurance markets. As the principal is one, and the agents are many, among the purposes of government intervention there could then be that of insuring parents against (their children’s) bad luck.

Reverting to principal-agent language, the principal would like the agents to choose the socially desirable \( e \) and, if fertility is endogenous, \( n \). Recalling that \( n \) is observable, but \( e \) is not, the agent must then be induced to take the desired level of the action by making \( \varphi \) conditional on \( h \). If fertility is endogenous, the desired \( n \) can be obtained by a forcing contract.

If fertility is exogenous, the principal chooses the function \( \varphi (.) \), i.e. a \( \varphi \) for each \( h \), so as to maximize (56), subject to the government budget constraint (60), and to the incentive-compatibility constraint (58).\(^{48}\) The first-order condition is that \( \varphi (h) \) must satisfy, for each \( h \),

\[
(U' - \lambda) \phi + \mu (-nw'U\phi + U'\phi_\mu) = 0,
\]

where \( \lambda \) is the marginal social utility of tax revenue, the Lagrange-multiplier associated with (60), and \( \mu \) the marginal social utility of relaxing the incentive-compatibility constraint, the Lagrange-multiplier associated with (58).

Suppose, for a moment, that the principal can either observe or infer \( e \). As the incentive-compatibility constraint is not binding \((\mu = 0)\), (61) reduces to \( U' = \lambda \). A first best can then be implemented by choosing the payment schedule \( \varphi (.) \) so that

\[
\varphi(h) + v(h) = \text{const.}
\]

This means that the first-best policy assures the agent a given level of utility, whatever the realization of \( h \). If parents do not care about the well-being of their children, and cannot get any money out of them, \((v \equiv 0)\), that implies a flat-rate child benefit scheme. By contrast, if parents care about their children’s welfare, or can get transfers from them, \((v'(h) > 0)\), (62) tells us that the government should insure parents against the risk that their children will do badly in life. In that case, the lower the realized value of \( h \), the higher \( \varphi \).

Let us now go back to the idea that \( e \) is not observable. Since the government must then depart from the full-insurance principle to satisfy

\(^{48}\) There is no need for a participation constraint because, with the number of children given, the agent cannot escape the government scheme by deciding not to have children.
the incentive-compatibility constraint, it will have to be content with a second best. It is convenient to re-write (61) as

\[
\frac{\lambda}{U'} = 1 + \mu \left( n r w' + \psi \right),
\]

where \( r \equiv -\frac{\phi'}{\phi} \) is the Arrow-Pratt measure of absolute risk aversion, assumed constant, and \( \psi(e) \equiv \frac{\phi'}{\phi} \) is a close relative of the likelihood ratio, assumed increasing in \( e \).\(^{49}\) Using standard arguments, it can be shown that \( \mu \) is positive, and

\[
\frac{d\varphi}{dh} = \frac{\phi'}{\lambda n r} - v'.
\]

Since \( \psi' \) and \( w' \) are positive by assumption, and \( \lambda \) and \( \mu \) must be positive at an optimum, (64) then tells us that, if parents do not care about their children’s future \( (v \equiv 0) \), the second-best child benefit rate is increasing in \( h \), \( \varphi'(h) > 0 \). Parents, in other words, should be rewarded for having children with high human capital, even though that is partly the result of chance. That may not be the optimal policy, however, if parents have an interest (for either altruistic or selfish reasons) in their children’s future earning ability. If \( v(.,) \) is increasing and concave (diminishing marginal utility of, or return on, \( h \)), the second-best child benefit rate could be U-shaped: decreasing in the child’s human capital at low levels of \( h \), where the insurance principle predominates; increasing at high levels of \( h \), where the incentive principle does.

If fertility is a choice variable, the government maximizes (56) with respect to \( \varphi(h) \), \( e \) and \( n \), subject to (60) – (58). Since the number of children is still visible, the government does not need to worry about (59) because it can fix \( n \) by a forcing contract:

\[
\Phi_1 = \varphi(h)n^* \text{ if } n = n^*,
\]

\[
\Phi_2 < \varphi(h)n^* \text{ if } n \neq n^*,
\]

where \( \Phi_2 \) is low enough to get parents to deliver the optimal (first or second-best) number of children, \( n^* \). The properties of \( \varphi(.,) \) are qualitatively the same as in the exogenous fertility case. The only difference endogenous fertility makes, compared with the exogenous fertility case, is that \( n \) is fixed at the optimal, rather than at an arbitrary level.

\(^{49}\)Both these assumptions are standard in the first-order approach adopted in this analysis,
6 Political acceptability

We now address the question whether a socially optimal system of public transfers, such as the one discussed in earlier sections, could be implemented in a democratic society. Browning (1975) makes the fundamental point that, since children do not vote, direct democracy will produce a pension system that is larger than the one which maximizes the lifetime utility of the representative agent. Indeed, since neither adults nor the old have a direct interest in children’s consumption, the public pension budget will be too large so long as children have no veto power. The argument is further developed in a long series of papers including, among others, Boadway and Wildasin (1989), Hansson and Stuart (1989), Verbon (1993), Peters (1995), Meijdam and Verbon (1996), Kolmar (1997), Grossman and Helpman (1998), and Kemnitz (2000); for an early survey, see Breyer (1994). Another series of papers, including Caillaud and Cohen (2000), and Boldrin and Rustichini (2000), re-proposes, at societal level, the question that Cigno (1993, 2000) had posed at the level of the family: could a constitution prescribing adult support for children and old people be self-enforcing? We examine this last aspect first, then look for voting equilibria.

A common feature of both strands of literature is that they take fertility as exogenous, and do not address the question of children’s survival into adulthood. Indeed, in both series of successor articles (not in Browning’s original contribution), children’s consumption is treated as a constant, and implicitly normalized to zero. Public transfers to children (or young people) come into the picture only insofar as they pay for education, and thus affect the productivity of future adults. That may seem curious, because current adults will need future ones to support them when they become old.\footnote{Even if they have accumulated capital (privately, or via a funded pension system), and do not require any form of personal assistance, they still need someone to man the machines.} Even if they are not altruistic, adults thus have an interest in children’s survival; productivity is a second-order consideration. In the same way as Samuelson implicitly assumed (according to Shubick) that parents are biologically programmed to look after their own children, so these authors implicitly assume that political agreement on legislation forbidding parents to let their children starve is reached as a matter of course.

6.1 A social compact?

Caillaud and Cohen (2000) tackle the problem of the possible existence, at societal level, of something equivalent to a family constitution within
a highly simplified framework. Population is assumed constant. Children, are not in the picture. Adults$^{51}$ produce, but do not consume,$^{52}$ a perishable consumption good. The old consume, but do not produce, that same good. Production at date $t$ is determined by

$$y^t = k^t l^t,$$  \hspace{1cm} (66)

where $l^t$ is the labour deployed by an adult born at $t$, and $k^t$ is interpreted as the state of knowledge (but could just as well be capital in the conventional sense) at that same date. The time-path of $k$ is exogenous, but nothing of substance changes if it is endogenized. The lifetime utility of generation $t$ is given by

$$U^t = -v (k^t, l^t) + c^{t+1},$$  \hspace{1cm} (67)

where $v (k^t, .)$ is a convex loss function, measuring the disutility of supplying labour (given the current state of knowledge) in the first period of a person’s life, and $c^{t+1}$ is consumption in the second period of life.

A Pareto-optimal allocation is characterized by choosing labour supply so as to maximize the utility of the representative agent, (67), subject to (66). The market alone will not yield such a solution. Given that people care only about their own consumption, as in the basic model used in earlier sections, generation $t$ will in fact produce income only if this induces generation $t + 1$ to do the same. In the absence of a mechanism ensuring this, nobody will produce anything.$^{53}$ Consequently, no generation will grow to be old. We are back to Samuelson (1958).

The way out proposed by Caillaud and Cohen is the same as in Cigno (1993, 2000). The authors look for the equivalent, for society as a whole, of a family constitution – a social compact (they use the expression ”standard of behaviour”) so designed that any ”generation should not be in a position such that it would prefer to erase the past, name itself generation [0] and reinitialize the strategy profile that was followed up to this date, rather than continue to abide by the current strategy profile” (Caillaud and Cohen, 2000). A social compact yielding a Pareto-optimal allocation of consumption meets this criterion, and is thus unamendable. Alternative approaches, such as the one proposed by Kotlikoff et al. (1986), who view the constitution as an asset, that the

$^{51}$The authors actually use the term ”young”. We say ”adult” for consistency with the terminology used so far.

$^{52}$A more palatable way of putting this would be to say that adult consumption is a constant (determined by subsistence requirements?), normalized to zero.

$^{53}$Or rather, young people will deploy the amount of labour, and produce the amount of goods, that just meets its own immediate (subsistence?) consumption requirements.
old would like to sell to the young, do not pin down a single standard of behaviour.

A problem with this transposition of the constitution idea from the family to society as a whole is that a single defector cannot be punished without also punishing the whole generation to which she belongs. Given that a person may be punished even if she complies, there is then a free-riding problem. The problem goes away only if adults are altruistic towards the old, as assumed in an earlier contribution by Veall (1986). Another problem is that each adult is supposed to know not only how her own parent behaved a period earlier, but also how every other member of her parent’s generation did. That imposes an unrealistically heavy informational requirement on the workability of the scheme.

6.2 Voting equilibria

Instead of looking for conditions that would make an intergenerational arrangement unchangeable, the public choice literature on public transfers starts from the premise that any decision taken by a parliament can be reversed by the next. The economy is modelled as a sequence of non-cooperative games. At any given date, the adults of the day choose current saving, taking current and future taxes and benefits as given. The government of the day chooses current taxes and benefits, taking current saving decisions, and future taxes and benefits, as given.

Since future taxes and benefits will be decided by the next government, the current Nash equilibrium is conditioned by expectations about what the next government will do. With the notable exception of Grossman and Helpman (1998), a limitation of this literature is that it does not take into account the impact of today’s political decisions on the scope for future ones. Boadway and Wildasin (1989) assume arbitrary expectations about future decisions, Meijdam and Verbon (1996), and Kemnitz (2000) impose rational expectations. Hansson and Stuart (1989) effectively assume the existence of a constitution giving each generation the right to block any new legislation that would leave it worse off.

An aim of this literature is typically that of predicting the effects of “population aging” (an exogenous drop in either fertility or mortality) on social security policy and economic growth. Since the models describe, rather than prescribe, the behaviour of politicians, social optimality and efficiency issues take second place. With the exception of Hansson and Stuart, who assume ascending altruism (from adults to old people) like Veall (1986), a common assumption is that individuals, and the government they elect, are self-interested. Consumers maximize the utility of their own consumption over what is left of their life cycle. The government maximizes the probability of re-election.
In contrast with Browning’s seminal contribution, which assumes direct democracy, another common feature of the public choice literature on intergenerational transfers is that it assumes a representative democracy. Unlike a direct democracy, where each single policy has to be approved by the electorate, a representative democracy is characterized by the fact that governments are voted-in on the basis of broad, often vaguely worded, electoral programmes. That gives them a certain latitude over which measures actually to implement, and leaves them open to pressure by interest groups (which, in our particular context, coincide with age groups, or generations).

There are two ways of modelling the political process. Becker (1983) makes the relative political weight of each interest group a function of its relative expenditure on lobbying. Coughlin (1986), by contrast, shows that maximizing the probability of re-election in a two-party system tantamounts to maximizing the sum of the objective functions of the voters. Coughlin et al. (1986) nuance this by introducing ideological bias in favour of one or the other party, and show that more ideologically homogeneous groups influence government policy more than less homogeneous ones.

The literature reviewed here draws on both these considerations by writing the government’s objective (some authors call it “target”, others “political support”) function, at any date $t$, as a weighted sum of the utilities of generations $t$ and $t-1$. This differs from a conventional social welfare function in that the relative weight of each generation depends on its ability to exert political influence, rather than on ethical considerations such as equity. As only electors count, children have zero political weight.

6.2.1 Voting over pensions

Meijdam and Verbon (1996) postulate a closed economy, so that the interest rate is endogenous. They prefer this to the more common “small open economy” assumption in order to avoid getting a solution with either positive saving and no transfers, or positive transfers and no saving. Capital is thus financed entirely by domestic saving.

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54 Coughlin (1986) speaks of “reputation”.
55 Children could still be an argument in the objective function of the government if, as in Cigno (1983, 1986, 2001), they were an argument in those of their parents, but it is assumed that they are not.
56 We know from the family-level model of sub-section 3.2 that a deterministic model with given interest rate does indeed have this property. But we also know that the property disappears, even if the interest rate is exogenous, if we introduce survival uncertainty as in Rosati (1996). Uncertainty about how many adults will be there to support us in old age may in fact generate precautionary saving.
At any date \( t \), adults choose \((c_1^t, c_2^t, s^t)\) so as to maximize (46), subject to
\[
c_1^t = w^t - \theta^t - s^t
\] (68) and
\[
c_2^t = s^t r^t + \eta^{t+1},
\] (69)
taking the current pension contribution, \( \theta^t \), and the future pension benefit, \( \eta^{t+1} \), as given. As usual, the first-order condition yields (6). Having conveniently assumed that children live on air, this condition tells us that consumption is efficiently allocated over the life-cycle of each generation. The old have no allocative decision to take. Given the current pension benefit, \( \eta^t \), their consumption at date \( t \) is determined by past saving decisions,
\[
c_2^{t-1} = s^{t-1} r^{t-1} + \eta^t.
\] (70)
Since \( k^t \) is pre-determined by \( s^{t-1} \), the private sector of the economy is closed using (1), (2) and (16).

Taking \( s^t \) and \( \eta^{t+1} \) as given, today’s government chooses \( \theta^t \) and \( \eta^t \) so as to maximize its objective function,
\[
W^t = n^{t-1} [u_1 (c_1^t) + u_2 (c_2^t)] + \rho^t u_2 (c_2^{t-1}),
\] (71a)
where \( \rho^t \) is the political weight of the old, subject to (68)-(70), and to the pay-as-you-go constraint,
\[
\eta^t = \theta^t n^{t-1}.
\] (72)
The political weight of the old could simply reflect their relative numerical strength, in which case \( \rho^t = 1 \) for all \( t \). More generally, it will reflect their ability to coordinate politically, and influence policy by lobbying.

Meijdam and Verbon assume that the political weight of the old increases with their relative number, \( \rho^t = \rho (n^{t-1}) \), \( \rho^t (.) < 0 \). As the same authors point out, however, the larger a group, the more costly it is for its members to coordinate their lobbying activities.\(^{57}\) The political weight of the old could thus decrease as their number increases, \( \rho^t (.) > 0 \). Casual observation suggests that an increase in the dependency ratio raises public concern for the welfare of the working generations, rather than of the retired.

\(^{57}\)Such a line of reasoning is followed in Kemnitz (2000); see the next sub-section.
The first-order conditions yield

$$\frac{u_1'(y^t - \theta^t - s^t)}{u_2'(s^{t-1}r^{t-1} + n^{t-1}\theta^t)} = \rho^t. \quad (73)$$

If a Nash equilibrium exists, the contribution level solving (73) maximizes the government’s chances of re-election. Will this sequence of political equilibria allocate consumption efficiently? The question is not addressed in the original paper, but we can take a stab at it exploiting the analogies with another paper by the same authors, Meijdam and Verbon (1997).

Given (6), (73) implies $r^t = \rho(n^{t-1})$ for all $t$. If the exogenously given rate of population growth is constant over time ($n^t = n$ for all $t$), the political process yields a steady state characterized by

$$r = \rho(n). \quad (74)$$

Interpreting $\rho$ as the social discount factor, the condition ("modified golden rule") for a social optimum in a closed economy with population growth rate lower than the interest rate,

$$n < r. \quad (75)$$

In conclusion, voting equilibria may support transfers to the old, and the resulting allocation may be dynamically efficient. Intuitively, the argument goes as follows. The old will always be happy to vote for a party that promises them a pension (paid for by others). If the old are not the majority ($n \geq 1$), however, such a party must get the vote of adult voters too. The latter will be in favour of a pension system is they expect to benefit from it in due course. Even if they do, however, there is no guarantee that the resulting allocation will be efficient. Given that no parliament can commit future ones, there is in fact nothing to ensure that the consumption allocation brought about by the contribution level, $\theta^t$, on which a political compromise was reached at any date $t$, is not Pareto-dominated by the one that would be brought about by a different contribution level. Since a different $\theta^{t+1}$ would require a different $\theta^{t+2}$, and so on, the electoral system will not produce the policy reform that would yield the Pareto-superior allocation.

58 In the notation of section 1, $\delta = \frac{1}{\rho}$.

59 In general, only some of the adults need to be in favour. But adults are assumed to be all the same, and thus to vote all in the same way.
6.2.2 Voting over pensions and educational subsidies

Finding that, if pensions are the only item on the agenda, the political process may not deliver a system of intergenerational transfers should not come as a surprise. Given a capital market, or the possibility of accumulating a durable good, adults can in fact do without public pensions, because they can save for old age. Finding that a pension system may not allocate consumption efficiently should not be a surprise either, because we know from sections 2 and 4 that an efficient system of intergenerational transfers includes support for children, as well as for the old.

Kemnitz (2000) brings back transfers to children in the form of educational grants. This increases the scope for intergenerational cooperation, because educational investment increases future per capita income, and thus makes it possible to increase pension benefits (of interest to adults, as well as to the old) without increasing contributions. Konrad (1995) produces a similar argument: the old have an interest in paying for public education because it shifts the Laffer curve of pension contributions. Mutatis mutandis, these two papers present similarities with the Cremer et al. (1992) model, reviewed in sub-section 4.1, where parents choose strategically how much to spend for their children’s education with an eye to how this will raise their pay-off in the subsequent bequests-for-attention game. There, however, the game is restricted to members of the same family. Here, it involves the whole polity.

Let $\varphi$ be again the amount that the government pays to parents for each of their children, and $\tau$ the lump-sum tax used to finance these transfers. As in the model of Section 4, we interpret $\varphi$ as an educational grant (again, children eat nothing), but we also assume that parents are obliged to set $e = \varphi$. In other words, education is not only free, but also compulsory. A pension system with benefit $\eta$, and contribution $\theta$ is still in place. Human capital is still determined by (48), but $\xi$ now stands for the parent’s stock of human capital (parents have a tutorial role), $\frac{\xi}{h^{t-1}}$, rather than for the child’s own native talent.

Beside putting child benefits on the political agenda, Kemnitz introduces uncertainty about survival into old age. Assuming a perfect annuity market, and denoting by $\pi$ the probability that an adult will

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60 The point that observable actions are controllable keeps coming up again and again. In an education context, we can think of traditional enforcement methods (police, truant catchers). In other contexts, we have seen that the same result can be achieved by a "forcing contract".

61 But, of course, either of these transfer systems could be inactive ($\theta$ or $\tau$ equal to zero).

62 Without it, there would be precautionary saving (to guard against the risk of
get to be old, each dollar saved by an adult at date $t$ now yields $\frac{r}{\pi}$, rather than simply $r$, a period later. Since uncertainty leaves scope for a solution with both saving and public transfers greater than zero even if the interest rate is exogenous, there is no need to assume a closed economy. The author does in fact use the ”small open economy” assumption encountered in earlier sections of this chapter.

Another innovation is that the political weight of each age group explicitly depends, à la Becker, on how much the group spends to influence government policy. Therefore, political weight is now truly endogenous. Since it benefits all members of the group equally, political weight is a kind of local (group-specific) public good. As all persons of the same age are the same, there is no problem of preference aggregation (decisions are unanimous), but there is still an enforcement problem. To get round this, it is assumed that ”influence” expenditure pays not only for lobbying, but also for maintaining group discipline (deter free riding). However, as the cost of maintaining discipline increases with numbers (like Cause’s transaction costs), the amount of political influence bought by a dollar decreases as the size of the group increases. That apart, the political process is still modelled as a non-cooperative game, with voters choosing saving and expenditures, and the government choosing policy.

Being uncertain whether they will still be alive at $t+1$, adults at date $t$ choose $(c^t_1, c^t_2, s^t, x^t_1, x^t_2)$ so as to maximize the expectation of (46),

$$E \left( U^t \right) = u_1 \left( c^t_1 \right) + \pi u_2 \left( c^t_2 \right),$$

subject to

$$c^t_1 = \left( w^t - \theta^t - \tau^t \right) h^t - x^t_1 - s^t$$

and

$$c^t_2 = \frac{s^t r^t}{\pi} + \eta^{t+1} - x^t_2,$$

where $x^t_i$ is ”influence” expenditure in period $i$ ($i = 1, 2$). As in Section 4, $w^t$ is interpreted as the rate of return to human capital, rather than the wage rate, at date $t$. The wage rate is again $h^t w^t$, but $h^t$ is now entirely determined by past education policies, rather than private educational investments. The private sector of the economy is closed by the factor pricing equations, (54) and (55).\(^{64}\)

\(^{63}\)Kemnitz uses a log-linear utility function to get explicit results.

\(^{64}\)Since parents cannot choose whether to invest in human capital or in other assets, there is no portfolio condition. Given that education is fully paid for by the government, there is no condition saying how much to borrow for education.
The government’s objective is

\[ W^t = n^{t-1} E(U^t) + \pi \rho^t u_2 \left( s^{t-1} r^{t-1} + \eta^t - x^t_{2-1} \right), \]  

(79)

where \( E(U^t) \) is given by (76) – (78). This differs from (71a) not only because survival into old age is now uncertain, but also because the weight attached to the old is now a function of ”influence” expenditures, as well as numbers,

\[ \rho^t = \rho \left( \frac{x^{t-1}_2}{x^t_1}, \frac{n^{t-1}}{\pi} \right). \]  

(80)

Under certain functional restrictions, Kemnitz demonstrates that a sub-game perfect Nash equilibrium exists. However, there is no guarantee that the resulting system of intergenerational transfers will be efficient.

7 Conclusion

We started by asking whether intergenerational cooperation \( (i) \) is socially desirable, \( (ii) \) will be realized by spontaneous agreement at some level. The answer to \( (i) \) is a clear yes. The answer to \( (ii) \) is problematic.

Individual optimization coordinated only by the market does not allocate consumption efficiently if individuals are self-interested; it may, but it is not likely, if individuals are altruistic. The existence of self-enforcing ”constitutional” arrangements delivering an efficient allocation of resources across generations can be demonstrated at the family level. If all members of the same generation are the same, however, fertility will be higher than the Millian optimum. If they are not, it is possible (but, again, not likely) that the aggregate fertility rate will be just right. The existence of a similar arrangement at societal level appears less plausible, because self-enforceability requires the assumption that everyone knows what everyone else does, and this becomes less and less tenable as the group gets larger.

We then asked whether, in the absence of decentralized mechanisms generating a socially optimal, or at least efficient, outcome, there are policies that would. The answer is yes. The policies considered are a public pension system, and a child benefit scheme. The outcome is a first best if the relevant personal characteristics and individual actions are visible, a second best if they are not. Where policies intended for the benefit of children require the intervention of parents, the problem has a principal-agent structure, with the government in the role of principal, and parents in that of agents.

Public transfers presuppose the existence of a political equilibrium in favour of such a policy. In the case of intergenerational transfers, this
may require an alliance between different generations. Voting models recognize that the policy introduced by one parliament can be reversed by the next. A sequence of temporary political equilibria may allocate resources efficiently across generations, but there is no guarantee that it will. Indeed, if transfers to children pay for consumption only, there is no reason to expect that self-interested adults and old people will vote in favour of such benefactions. By contrast, transfers to the old could be too large. The likelihood that a parliament will legislate transfers to children increases, if these transfers serve to pay for an investment (education) which raises the contributive capacity of tomorrow’s adults, and will thus potentially benefit tomorrow’s old people.

8 References


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