ABSTRACT

Racial Identity and Education*

We investigate the sources of differences in school performance between students of different races by focusing on identity issues. We find that having a higher percentage of same-race friends has a positive effect on white teenagers’ test score while having a negative effect on blacks’ test scores. However, the higher the education level of a black teenager’s parent, the lower this negative effect, while for whites, it is the reverse. It is thus the combination of the choice of friends (which is a measure of own identity) and the parent’s education that are responsible for the difference in education attainment between students of different races but also between students of the same race. One interesting aspects of this paper is to provide a theoretical model that grounds the instrumental variable approach used in the empirical analysis to deal with endogeneity issues.

JEL Classification: A14, I21, J15 and J24
Keywords: education achievement, endogeneity issues, ethnic minorities and peer effects

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1 Introduction

The educational experience of American school students is a multifaceted phenomenon that encompasses far more than academic achievement. Other important aspects of the educational experience include daily participation in school as well as students’ feelings about their school. These latter are much less studied and thus understood by economists, even though these aspects of the educational experience also have important consequences on drop out rates, delinquency activities and school performance. In particular, economists have been trying to explain education attainment differences between blacks and whites (see, e.g., the recent survey by Neil, 2006) by putting forward the role of peer effects, school quality, neighborhood effects, etc. Less research has been however devoted to explaining education differences within a race and we believe that the “sociological aspects” in education mentioned above can help us in this task. For example, if one controls for different characteristics of the parents (e.g. human capital), the neighborhood (e.g. segregation) and the school quality (e.g. teacher/student ratio, average test scores), one still find different school performances for black and white students. The natural explanation for this is peer effects. For example, Hoxby (2000) find that students are affected by the achievement level of their peers and that peer effects are stronger *intra*-race. We go further in this direction by trying at a theoretical and empirical level to ascertain the effects of ethnic identity (here choice of same-race friends) on school performances of black and white students.

Ethnic identity is obviously a difficult and complex question. Akerlof and Kranton (2000) define it as a person’s sense of self or self image. A person’s sense of self or self image is then said to make his or her identity in that “his or her identity is bound to social categories; and individuals identify with people in some categories and differentiate themselves from those in others.” (page 720). In other words, self-image, or identity, is associated with the social environment; example of social categories include racial and ethnic designations. Pursuing this idea, Akerlof and Kranton (2002) try to link together identity and schooling. A student’s primary motivation is his or her identity and the quality of the school depends on how students fit in a school’s social setting. In the context of schools, social categories could be, for example, “jock” and “nerd”. In particular, it has been observed that African Americans tend to “choose” to adopt what are termed “oppositional” identities, that is, some actively reject the dominant ethnic (e.g., white) behavioral norms while others totally assimilate to it (see, in particular, Ainsworth-Darnell and Downey, 1998). Studies in the US have found, for example, that African American students in poor areas may be ambivalent about learning standard English and performing well at school because this may be regarded as “acting white” and adopting mainstream identities (Wilson, 1987, Delpit, 1995, Fordham and Ogbu, 1986, Ogbu, 1997, Austen-Smith and Fryer, 2005, Battu et al., 2005, Selod and Zenou, 2006). In

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1 Sociologists have been more interested by such issues. See e.g. Coleman, 1961, or more recently, Kirkpatrick Johnson et al. (2001). Akerlof and Kranton (2002) survey some of these issues from the noneconomic literature and propose to “translate” them into an economic model of students and schools.
some instances, oppositional identities produce significant economic and social conflicts.

In the present paper, we pursue this line of research by investigating the impact of “identity” on school performance for black and white students. In our model, each adolescent chooses the percentage of same-race friends as well as how much effort he or she puts in education. Choosing the percentage of same-race friends is an indication of identity, especially for blacks. Interestingly, we observe in the data that blacks tend to have “oppositional identities” since white students tend to mostly have white friends while blacks have both black and white friends. We show that having a higher percentage of same-race friends has a positive effect on white teenagers’ test score while having a negative effect on blacks’ test scores. However, the higher the education level of a black teenager’s parent, the lower this negative effect, while for whites, it is the reverse. It is thus the combination of the choice of friends and the parent’s education that are responsible for the difference in education attainment between students of different races but also between students of the same race. We then bring the model to the data. The main issue in the empirical strategy is that the choice of same-race friends is endogenous. In the theoretical analysis, which explicitly models this choice, the key exogenous mechanism driving the choice of the race of a (black) student’s friends is the individual’s attachment to his/her culture of origin, referred as the sense of black identity. We translate this framework into the empirical analysis. Firstly, we focus mainly on blacks since, in the model, it is shown that whites’ choice of same-race friends is only slightly affected by their identity. Secondly, we undertake a two-stage instrumental variables procedure, where, in the first stage, we estimate the likelihood to have black friends with appropriate instruments (measuring ethnic identity), while, in the second stage, we estimate the impact of choice of same-race friends on school performance, using the predicted values from the first stage estimation.

Our empirical investigation exploits a unique and very detailed data set of friendship networks within US school (the National Longitudinal Study of Adolescent Health), which allows us to construct an objective measure of individuals’ attachments to their culture of origin on the basis of the number of same-race friends nominated by each student as his/her best friends. Moreover, the detailed information contained in our data on the ethnicity of the parents (interracial marriage) and on various aspects of religious participation of the adolescent, i.e. religious affiliation, importance of religion, religious service attendance, involvement in church youth groups and fanatical faith attitude (e.g. thinking of being born-again-Christian) provides the appropriate instrumental variables to identify the first step ethnic preferences equation.

In both cases (with and without instruments), we find that the predictions of the model are confirmed by our empirical results. The choice of same-race friends is strongly affected by blacks’ ethnic identity as measured by religious activities and interracial marriage. This, in turn, affects school performance in the sense that the higher the percentage of black friends a black adolescent has, the lower the test scores he or she achieves.

The remainder of the paper is structured as follows. The next section exposes the theoretical
model, first when the choice of friends is exogenous and, then, when it is endogenous. Section 3 describes the empirical strategy, data and the definition of our key variables and provide some descriptive evidence on ethnic preferences. Sections 4 and 5 present the empirical results, differentiating the direct estimation strategy (section 4) from the two-stage instrumental variable procedure (section 5). Finally, section 6 concludes. All proofs of the theoretical model are relegated to Appendix 1.

2 Theoretical model

We focus on black and white adolescents. A model in two parts will be proposed. In the first one, we consider the choice of friends as exogenous and we then endogeneize it.

2.1 Exogenous choice of friends

There are two groups of individuals, blacks (indexed by \( j = B \)) and whites (indexed by \( j = W \)). In this section, there is only one main difference between blacks and whites. We assume that, on average, whites’ parents have a higher human capital (or education) level than blacks’ parents, i.e. 

\[ k_W > k_B \]  

\( (H1) \)

This is a widely documented fact (see e.g. Neal, 2006) and it is also true in our data (see Table 1 below). The utility for individual \( i \) belonging to group \( j = B, W \) (referred to as individual \( ij \)) is given by:

\[ U_{ij}(y_{ij}) = y_{ij}^\beta k_{ij} - \frac{1}{2}y_{ij}^2 \]  

\( (1) \)

where \( y_{ij} \) the level of school performance (i.e. test score) chosen by individual \( i \) (i.e. how much effort he/she puts in his/her studies), \( k_{ij} \) is the human capital level of individual \( ij \)’s parent, and \( \bar{k}_{ij} = x_{ij}\bar{k}_{-iB} + (1-x_{ij})\bar{k}_{-iW} \) is the average human capital level of the parents of \( ij \)’s friends. The latter is a weighted average of the human capital level of the parents of individual \( ij \)’s black and white friends. In this formulation, \( 0 \leq x_{ij} \leq 1 \) denotes the percentage of black friends chosen by individual \( ij \), while \( \bar{k}_{-iB} \) and \( \bar{k}_{-iW} \) are respectively the average parents’ human capital of the black and white friends of \( ij \) (but not of \( ij \) himself/herself). We assume that \( 0 < \beta < 1 \) to ensure concavity. Also, for simplicity, we assume that the average parents’ human capital of \( ij \)’s black friends is the same as the average human capital of black parents in the population, i.e. \( \bar{k}_{-iB} = \bar{k}_B \). We assume the same for \( ij \)’s white friends, i.e. \( \bar{k}_{-iW} = \bar{k}_W \). As a result,

\[ \bar{k}_{ij} = x_{ij}\bar{k}_B + (1-x_{ij})\bar{k}_W \]  

\( (2) \)

One has to notice the difference between \( y_{ij} \) and \( k_{ij} \). The former is the effort exerted by the teenager \( ij \) when studying, i.e. how much time he/she spends studying, doing his/her homework, etc. In

\footnote{This assumption is not crucial to obtain our main results. It just simplifies the exposition.}
the empirical analysis, we measure \( y_{ij} \) by the test score of individual \( ij \) assuming that effort and outcome are highly correlated. The later, \( k_{ij} \), is the human capital of \( ij \)'s parent.

Let us now explain in more details this utility function. It has two parts. As in Borjas (1992), the first part shows the positive impact of the human capital of the parent, \( k_{ij} \), and of parents’ friends, \( \overline{k}_{ij} \) on own utility. In particular, the average human capital of the parents of \( ij \)'s friends has an external effect on the utility function. The second part is the cost of providing effort, which is, as usual, assumed to be increasing and convex.

Each individual \( ij \) chooses \( y_{ij} \) that maximizes (1), where \( \overline{k}_{ij} \) is given by (2). We have the following result.

**Proposition 1** Assume that the choice of friends is exogenous and that \((H1)\) holds. Then,

(i) the more black teenagers have black friends, the lower their school performance whereas the more white teenagers have white friends the higher their school performance;

(ii) for any individual, black or white, the higher the human capital of their parent and/or the higher the human capital of their parents’ friends, the higher their school performance;

(iii) the higher the education level of a black teenager’s parent, the lower the negative effect of same-race friends on his/her school performance;

(iv) the higher the education level of a white teenager’s parent, the higher the positive effect of same-race friends on his/her school performance.

These results are interesting though easy to understand. The more black adolescents hang up with same-race friends, the more likely their parents’ friends have low human capital and thus the lower their education outcome \( y_{iB} \). For whites, it is exactly the contrary since white parents have on average a higher human capital level than black parents. The cross effects described in (iii) and (iv) in Proposition 1 are more subtle. If individual \( iB \)'s parents are highly educated then the negative effect of having black friends can be reduced. This is because both \( k_{iB} \) and \( \overline{k}_{iB} \) have a positive external effect on individual \( iB \)'s education outcome.

Figure 1 illustrates this result by showing the returns to education of having white friends. Once can see that, even if blacks and whites have exactly the same percentage of white friends, which means that the human capital of their parents’ friends is the same, i.e. \( \overline{k}_{iB} = \overline{k}_{iW} = \overline{k}_i \), white teenagers obtain a higher level of education compared to blacks because the human capital of their parents \( k_{iW} \) is greater than that of blacks \( k_{iB} \). In particular, using (17) and (16) in Appendix 1, one can calculate:

\[
\Delta y = y_{iW}^* - y_{iB}^* = \beta_1^{1/(2-\beta_1)} \overline{k}_i^{1/(2-\beta_1)} \left[ k_{iW}^{1/(2-\beta_1)} - k_{iB}^{1/(2-\beta_1)} \right]
\]
which implies that $\Delta y$ is increasing in the difference between $k_{iW}$ and $k_{iB}$. Of course, in our model, we assume that, on average, whites have a higher human capital than blacks, which means that, for some workers, the returns to education can be higher for blacks. Finally, with Figure 1, one can easily understand the cross-derivative results of parts (iii) and (iv) of Proposition 1. If we rewrite these results in terms of percentage of white friends (instead of same-race friends), then they state that

$$\frac{\partial^2 y^*_i}{\partial (1-x_{iB}) \partial k_{iB}} > 0 \quad \text{and} \quad \frac{\partial^2 y^*_i}{\partial (1-x_{iW}) \partial k_{iW}} > 0$$

Thus, a higher parent’s human capital will shift upward each curve in Figure 1 so that the returns from having white friends will be even greater. Of course, we can proceed exactly the same way and derive opposite results on the negative returns to education from having black friends.

[Insert Figure 1 here]

2.2 Endogenous choice of friends: The two-stage model

Let us now model the choice of friends as a first stage of a two-stage process where adolescents first choose their friends, i.e. $x_{ij}$, and then their education effort $y_{ij}$. In terms of notation, $\tilde{x}_{ij}$ denotes the percentage of same-race friends of individual $ij$, that is $\tilde{x}_{iB} = x_{iB}$ for blacks and $\tilde{x}_{iW} = 1 - x_{iW}$ for whites. The second stage is already solved and the outcomes are given by (see equations (17) and (16) in Appendix 1):

$$y^*_i = \beta_1^{1/(2-\beta_1)} k_{iB}^{1/(2-\beta_1)} [\tilde{x}_{iB} \tilde{k}_B + (1 - \tilde{x}_{iB}) \tilde{k}_W]^{1/(2-\beta_1)} \quad (3)$$

for blacks, and

$$y^*_i = \beta_1^{1/(2-\beta_1)} k_{iW}^{1/(2-\beta_1)} [(1 - \tilde{x}_{iW}) \tilde{k}_B + \tilde{x}_{iW} \tilde{k}_W]^{1/(2-\beta_1)} \quad (4)$$

for whites. Let us now describe the first stage where individual $ij$ choose his/her friends $x_{ij}$. Each individual $i$ belonging to race $j = B, W$ (refers to as individual $ij$) has the following utility function:

$$V_{ij}(\tilde{x}_{ij}) = z_{ij} \tilde{x}_{ij} - \frac{1}{2} b \tilde{x}_{ij}^2 + d y_{ij}(\tilde{x}_{ij}) \quad (5)$$

where $0 \leq \tilde{x}_{ij} \leq 1$ denotes the percentage of same-race friends chosen by individual $ij$, $d > 0$ is a parameter that describes how much teenagers value education, $b > 0$, and $y_{iB}(x_{iB}) = y^*_i$ and $y_{iW}(\tilde{x}_{iW}) = y^*_i$ are defined by (3) and (4), respectively. The variable $z_{ij} > 0$ is a measure of the strength of own identity. For blacks, for example, $z_{iB}$ could capture how important is black identity for individual $ij$ (like for example how involved are the parents or the child in activities that strengthen identity). We will discuss extensively the empirical counterpart of $z_{ij}$ in the empirical analysis below. Observe that, if education attainment did not play any role in this utility function, then a black teenager $i$ would choose $\tilde{x}_{iB} = x_{iB} = z_{iB}/b$ while a white teenager $i$ would choose $\tilde{x}_{iW} = 1 - x_{iW} = z_{iW}/b$. In that case, the higher $z_{iW}$, the more likely teenagers choose friends of same race.
Quite naturally, we would like to assume that, a priori, (i.e. without any consideration for education attainment), a black and a white adolescent always prefers to interact with friends of the same race. For that to be true, we assume that

\[
\frac{z_iB}{b} > \frac{1}{2} \text{ and } \frac{z_iW}{b} > \frac{1}{2}
\]

which just implies that, if education would not matter, blacks and white teenagers would always choose to have at least fifty percent of same-race friends, i.e. \( \tilde{x}_{iB} = x_{iB} > 1/2 \) and \( \tilde{x}_{iW} = 1 - x_{iW} > 1/2 \). The assumption \((H2)\) that, a priori, blacks prefer to interact with blacks while whites prefer whites is well documented in economics (see e.g. Cutler et al., 1999) but also in sociology and psychology where adolescents feel more comfortable around others of their own race (Clark and Ayers, 1988; Hallinan and Tuma, 1979). This does not mean that, in equilibrium, each of them will choose to exclusively have friends of the same race but it is just embedded in their preferences.

We first solve each individual’s problem separately. Let us first study the case of black adolescents. We have the following notation:

\[
K_B \equiv \alpha^{\beta_1/(2-\beta_1)} k_{iB}^{1/(2-\beta_1)} (k_W - \bar{k}_B) > 0
\]

\textbf{Proposition 2} Assume that \((H1)\) holds. Then, (i) If \(b/2 < z_iB < b\), three cases emerge:

(i1) If

\[
z_iB < K_Bk_{iB}^{(\beta_1-1)/(2-\beta_1)}
\]

each black teenager \(i\) only chooses friends who are whites, that is \(x_i^* = 0\), and his/her education attainment is given by:

\[
y_i^* = \beta_1^{1/(2-\beta_1)} (k_{iB} k_W)^{1/(2-\beta_1)}
\]

which is a positive function of his/her parent’s education \(k_{iB}\) and the average parent’s education of his/her white friends \(k_W\).

(i2) If

\[
K_Bk_{iB}^{(\beta_1-1)/(2-\beta_1)} \leq z_iB \leq K_Bk_{iB}^{(\beta_1-1)/(2-\beta_1)}
\]

then there is an interior solution where each black teenager \(i\) chooses a mix of black and white friends, i.e. \(0 < x_i^{(i2)} < 1\).

(i3) If

\[
z_iB \geq K_Bk_{iB}^{(\beta_1-1)/(2-\beta_1)}
\]

then there is an interior solution where each black teenager \(i\) chooses a mix of black and white friends, i.e. \(0 < x_i^{(i3)} < 1\), with \(x_i^{(i3)} > x_i^{(i2)}\).
(ii) If \( z_{iB} \geq b \).

\( (ii1) \) If

\[
z_{iB} > K_B \overline{k}_{iB}^{(\beta_1 - 1)/(2 - \beta_1)} + b
\]

each black teenager \( i \) only chooses friends who are blacks, that is \( x_{iB}^* = 1 \), and his/her education attainment is given by

\[
y_{iB}^* = \beta_1^{1/(2 - \beta_1)} \left( k_{iB} \overline{k}_B \right)^{1/(2 - \beta_1)}
\]

which is a positive function of his/her parent’s education \( k_{iB} \) and the average parent’s education of his/her black friends \( \overline{k}_B \).

\( (ii2) \) If \((7)\) holds, each black teenager \( i \) only chooses friends who are whites, that is \( x_{iB}^* = 0 \), and his/her education attainment is given by \((8)\).

\( (ii3) \) In all other cases, that is when both conditions \((9)\) and \((7)\) do not hold, there is an interior solution where each black teenager \( i \) chooses a mix of black and white friends, i.e. \( 0 < x_{iB}^* < 1 \).

There is a trade off between friends and education. On the one hand, a black adolescent would like to have as many black friends as possible, but, on the other, he/she values education and knows that, on average, whites’ parents have a higher education than blacks’ parents so that white friends are also valuable. So he/she is choosing the percentage of black and white friends anticipating its impact on education. In case \((ii1)\) where the black teenager chooses to have only black friends, condition \((9)\) holds if \( z_{iB} \) (the strength of black identity) is high enough or \( k_{iB} \) (the human capital of his/her parent) or \( d \) (how much he/she values education) are low enough. This is very intuitive since if a black teenager has a strong identity of being black, or his/her parent is not very educated so that his/her own returns to education are not high, or does not value very much education, then there is no point in having white friends and thus she/he chooses to hang up only with black friends. Of course, his/her education attainment will not be very high but will still depend on the education level of his/her parent and his/her friends’ parents. Cases \((i1)\) and \((ii2)\) predict the opposite and the intuition runs exactly the opposite way. If the identity of being black does not matter very much, or if his/her parent is very educated, or he/she values very much education, then having white friends is very important and this black teenager will choose to have only white friends. Finally, all the other cases show that for intermediate values of \( z_{iB} \), \( k_{iB} \) and \( d \), black teenagers choose a mixed of black and white friends. This choice is positively affected by \( z_{iB} \) and negatively by \( k_{iB} \) and \( d \).

In our data set,\(^3\) we find exactly these three behaviors from black adolescents. Figure 2 displays the empirical distribution of teenagers by share of same-race friends for blacks and whites, respec-

\(^3\)See section 3 below for a description of our data set.
tively. This figure shows that blacks are more heterogeneous in their choice of friends than whites since there are three modes in the distribution of friends: 32 percent of black teenagers have 80 percent of their friends who are blacks, 25 percent have 70 percent of their friends who are whites and 10 percent have 60 percent of their friends who are blacks (and thus 40 percent are whites). Cutler et al. (1999) find similar attitudes among blacks. They use the following question of the General Social Survey in the United States in 1982: “If you could find the housing that you would want and like, would you rather live in a neighborhood that is all black; mostly black; half black; half white; or mostly white?” On average, 67 percent of blacks choose either the third or fourth option, meaning that a large fraction of blacks would like to interact with whites (either because they like whites or because they anticipate the positive effects on education and labor market outcomes) but also 33 percent of them would like to interact mostly with blacks.

Figure 3a illustrates one of the possible interior equilibria, i.e. $0 < x_{iB}^* < 1$. For a black teenager, the increasing curve $g(.)$ indicates the marginal cost (in terms of education) of having black friends while the decreasing line $f(.)$ shows the marginal gain (in terms of identity) of having black friends. The intersection of these two curves gives the optimal choice of percentage of black friends for a black adolescent.

\[K_W \equiv d \frac{\beta_1^{1/(2-\beta_1)}}{(2-\beta_1)} k_{iW}^{1/(2-\beta_1)} (\overline{k}_W - \overline{k}_B) > 0 \quad (11)\]

**Proposition 3** Assume that (H1) holds. Then,

(i) if $b/2 < z_{iW} < b$, three cases emerge:

(i1) If

\[z_{iW} > b - K_W \overline{k}_W^{(\beta_1-1)/(2-\beta_1)} \]

then each white teenager \( i \) only chooses friends who are whites, i.e. \( x_{iW}^* = 1 - x_{iW}^* = 1 \), and his/her education attainment is given by:

\[y_{iW}^* = \beta_1^{1/(2-\beta_1)} (k_{iW} \overline{k}_W)^{1/(2-\beta_1)} \quad (12)\]

which is a positive function of his/her parent’s education $k_{iW}$ and the average parent’s education of his/her white friends $\overline{k}_W$.

\[4\text{The graph gives on the horizontal axes the share } l \text{ of same race friends (in total number of nominated friends) from 0.1 to 1 and in the vertical axes the percentage of teenagers having a share of same race friends between } l 	ext{ and } l-1.\]
\[(i2)\] If
\[b - K_W \frac{\beta_1}{(2 - \beta_1)} \leq z_i \leq b - K_W \frac{\beta_1}{(2 - \beta_1)}\]
then each white teenager \(i\) chooses a mix of white and black friends, i.e. \(0 < \tilde{x}_i^{(i2)} < 1\), but with \(\tilde{x}_i^{(i2)} > \max\left\{1/2, z_i/b\right\}\), implying that he/she has more than 50 percent of white friends.

\[(i3)\] If
\[z_i < b - K_W \frac{\beta_1}{(2 - \beta_1)}\]
then each white teenager \(i\) chooses a mix of white and black friends, i.e. \(0 < \tilde{x}_i^{(i3)} < 1\), with \(\tilde{x}_i^{(i3)} > \tilde{x}_i^{(i2)}\), implying that he/she has more than 50 percent of white friends.

\[(ii)\] If \(z_i \geq b\), each white teenager \(i\) only chooses friends who are whites, that is \(\tilde{x}_i^* = 1 - x_i^* = 1\).

This result is quite intuitive. Indeed, if \(z_i \geq b\), whites a priori prefer to have only white friends and since, on average, the parents of these white friends have a higher education level than black parents, then whites will never choose to have black friends. If, on the contrary, \(\frac{1}{2} < \tilde{x}_i < 1\), whites have less extreme preferences and may, in some cases, choose a mix of black and white friends, but always more than 50 percent of white friends. These cases are when education is not very valued and/or parents are not very educated. This is quite realistic since, in our data set, most white adolescents have white friends but some still have black friends. Indeed, Figure 2 shows that 65 percent of white pupils have more than 80 percent of same-race friends. Not surprisingly, we also find that the higher the human capital of \(iW\)’s parents and his/her parents’ friends, the higher his/her education attainment. These are the standard positive externalities in the education process highlighted for example in Borjas (1992). Figure 3b illustrates one of the possible interior equilibria, i.e. \(0 < x_i^* < 1\), when \(\frac{1}{2} < \tilde{x}_i^* < 1\).

[Insert Figure 3b here]

Let us study the properties of each optimal choice by doing a simple comparative statics analysis. We focus on blacks since the analysis is similar for whites.

**Proposition 4** Assume that \((H1)\) holds and assume that there is an interior solution for a black teenager \(i\) denoted by \(0 < x_i^* < 1\). Then, the higher \(z_i\) (the strength of black identity) or the lower \(k_i\) (the human capital of the parent) or the lower \(d\) (the taste for education), the higher the percentage of black friends and the lower the educational achievement \(y_i^*\). Also, if \(z_i\) is not too high or \(d\) is large enough, then the higher the education level of a black teenager’s parent, the lower the positive effect of \(z_i\) on \(x_i^*\) the percentage of same-race friends. Concerning the cross-derivative \(\frac{d^2 y_i^*}{d z_i d k_i}\), the sign is indeterminate.
Firstly, and not surprisingly, we are able to show that a higher human capital of the parent or a higher taste for education lead to a higher educational attainment. The new result here is the fact that $z_{iB}$, the strength of black identity, has a negative impact on $y^*_iB$, the educational attainment of a black adolescent. Indeed, the more a black teenager think that his/her black identity is important, the more he/she will choose black friends, the lower will be his/her educational achievement because of the peer group effect of the human capital of the parents of his/her friends. What is crucial here is that $z_{iB}$ does not affect directly education $y^*_iB$, but only indirectly through the choice of black friends $x^*_iB$, that is:

$$\frac{\partial y^*_iB}{\partial z_{iB}} = \frac{\partial y^*_iB}{\partial x^*_iB} \frac{\partial x^*_iB}{\partial z_{iB}}$$

and this effect is negative because $\frac{\partial y^*_iB}{\partial x^*_iB} < 0$ and $\frac{\partial x^*_iB}{\partial z_{iB}} > 0$.

Secondly, the other interesting result is the cross effect of $\frac{\partial^2 y^*_iB}{\partial z_{iB} \partial k_{iB}}$. Indeed, as in the case of exogenous $x^*_iB$, one would like to know the impact of $k_{iB}$ on the negative effect of $z_{iB}$ on $y^*_iB$. Intuitively, we would think that the higher the human capital of $iB$’s parent, the lower the negative impact of identity on education achievement. But because there are many effects in the theoretical model (indeed there are both direct and indirect effects of $k_{iB}$ on $\frac{\partial y^*_iB}{\partial z_{iB}}$), the sign is indeterminate. We will evaluate this impact in the empirical analysis and see which sign prevails.

We have determined the optimal choice of a black and a white teenager. It is quite natural now to ask if these choices are compatible in equilibrium. This is a difficult question and we provide an example where this is true. There are $N_B$ blacks and $N_W$ whites, with $N = N_B + N_W$. We focus on a symmetric equilibrium where all blacks make the same choice $x^*_iB = x^*_B$ and $y^*_iB = y^*_B \ \forall i = 1, ..., N_B$ and all whites also make the same choice $x^*_iW = x^*_W$ and $y^*_iW = y^*_W \ \forall i = 1, ..., N_W$. This is the case when all blacks’ (whites’) parents have the same human human capital $k_{iB} = k_B$ ($k_{iW} = k_W$) and all black (white) teenagers have the same sense of identity $z_{iB} = z_B$ ($z_{iW} = z_W$).

At the aggregate level, it is however very difficult to write a condition that guarantees that the different choice of friends are compatible because one has to keep tract of all the relationships. One could in principle derive an explicit network model (as in Jackson, 2006) where all friend relationships are explicitly modelled and reciprocal (undirected graph). In that case, by definition, all choices will be mutually consistent. But this would be extremely complicated and beyond the scope of this paper. We can however illustrate when our equilibrium concept works using a very simple example of a network where each individual has three friends and $N_B = N_W = 3$. In Figure 4, each black individual $B$ has two black friends and one white friend, i.e. $x^*_iB = 2/3$, while each white individual $W$ has two white friends and one black friend, i.e. $x^*_iW = 1 - \frac{2}{3} = 1/3$. In that case, the equilibrium is symmetric and a simple condition that guarantees that friend choices are mutually consistent is: $x^*_B + x^*_W = 1$, which means that the percentage of black friends chosen by blacks and whites has to sum to one. But this condition cannot be generalized to more general settings, unless extreme assumptions are adopted. To conclude, as long as whites do not have
extreme choices, that is for some parameter values, they choose to have black friends (which is true in cases (i2) and (i3) in Proposition 3), then it is reasonable to assume that one can always write a condition that guarantees that these choices of friends of blacks and whites are compatible with each other.

The aim of the theoretical section was to provide the main mechanisms that explain how the choice of same-race friends affects educational outcomes of black and white students. Let us now test this model.

[Insert Figure 4 here]

3 Empirical strategy and data

Empirical strategy We proceed in two steps. In the first one, as in section 2.1, we consider the choice of friends as exogenous and thus test directly the impact of “choice of friends of same race”, \( \tilde{x}_{ij} \), on educational outcomes of black and white teenagers, \( y_{ij}^* \). We also test the cross effects \( \frac{\partial^2 y_{ij}^*}{\partial x_{ij} \partial k_{ij}} \). In short, we test the results of Proposition 1.

To be more precise, we consider the following econometric equation, which is a linear reduced form of equations (3) and (4) for blacks and whites, respectively.\(^5\)

\[
y_{ij} = \alpha_{0j} + \alpha_{1j} \tilde{x}_{ij} + \alpha_{2j} k_{ij} + \alpha_{3j} \bar{k}_{ij} + \alpha_{4j} \tilde{x}_{ij} k_{ij} + \sum_{k=1}^{K} \beta_{kj} v_{ij}^k + \varepsilon_{ij}, \quad i = 1, \ldots, N, \quad j = B, W
\]

(Model 1)

where, as in the theoretical model, the dependent variable, \( y_{ij} \), is educational achievement of individual \( ij \), \( \tilde{x}_{ij} \) is the percentage of same-race friends for individual \( ij \), that is \( \tilde{x}_{ij} = x_{iB} \) for blacks and \( \tilde{x}_{ij} = 1 - x_{iW} \) for whites, \( k_{ij} \) denotes the human capital level of individual \( ij \)'s parent, \( \bar{k}_{ij} \) is the average human capital level of individual \( ij \)'s friends’ parent, \( v_{ij}^k \) (for \( k = 1, \ldots, K \)) is a set of \( K \) control variables containing an extensive number of individual, network, family, school and residential neighborhood characteristics, and, finally, \( \varepsilon_{ij} \) is a white noise error term. The exact empirical counterpart of each target variable is discussed in the next section. A precise description of all the control variables is contained in Appendix 2.\(^7\) Following the model, the predicted signs for

---

\(^5\)Because the use of cross-equation restrictions is rejected by our data, we perform a separate estimation for each group.

\(^6\)Observe that we do not report in (Model 1) the cross effect \( \tilde{x}_{ij} \bar{k}_{ij} \), which is present in equations (3) and (4). This is because we want to focus on the variables highlighted in the propositions. Including this term in (Model 1) does not change qualitatively the empirical results, which still support the predictions of the model. These results are available upon request.

\(^7\)Observe that the control set also includes the percentage of blacks in each child’s residential neighborhood, as an indicator of exposure of race \( j \) students to blacks (see the segregation literature, like e.g. Massey and Denton, 1993, or, more recently, Card and Rothstein, 2005), and the percentage of black teachers in the school. By doing so, we
\( \alpha_1B, \alpha_2B, \alpha_3B \) and \( \alpha_4B \) are respectively negative, positive, positive and negative, and the predicted signs for \( \alpha_1W, \alpha_2W, \alpha_3W \) and \( \alpha_4W \) are respectively positive, positive, positive and positive.\(^8\)

Let us now analyze the second step of our empirical analysis. As in section 2.2, we will not anymore consider the choice of friends as exogenous. From an empirical point of view, it is clear that there is an endogeneity problem when one wants to evaluate the impact of \( \tilde{x}_{ij}^* \) on \( y_{ij}^* \). We tackle this issue using a two-stage instrumental variables strategy, which is theoretically modelled in Section 2.2 and also supported by the descriptive evidence of our sample. We obtain a striking match between theoretical model, descriptive statistics, empirical strategy and estimation results.

First, because of Proposition 3 and Figure 2, where it is shown that in most cases white adolescents tend to have mostly white friends, we focus only on the choices of black teenagers and thus test Propositions 2 and 4. Second, the theoretical model provides an instrument for \( \tilde{x}_{ij}^* \), which is the strength of black identity, \( z_{iB} \) (we will discuss its empirical counterpart in the next section). Indeed, as showed by (13), \( z_{iB} \) directly affects \( \tilde{x}_{iB}^* \) but affects \( y_{iB}^* \) only through \( \tilde{x}_{iB}^* \). So, in order to test Propositions 3 and 4, our two-stage instrumental variables procedure will be as follows. In the first stage, we estimate the likelihood to have black friends with appropriate instruments, i.e., we estimate the impact of \( z_{iB} \) on \( \tilde{x}_{iB}^* \). In the second stage, we estimate Model 1 using the predicted values of \( x_{iB}^* \) from the first stage estimation instead of the \( \tilde{x}_{iB} \) itself. The predicted signs should be as in Proposition 4. Indeed, in the first stage, we should observe a positive and significant impact of \( z_{iB} \) on \( \tilde{x}_{iB}^* \) while, in the second stage, we should have the same predicted signs as in Model 1 for blacks.

**Data and definition of the variables**  Our data source is the National Longitudinal Survey of Adolescent Health (AddHealth), containing detailed information on a nationally representative sample of 90,118 pupils in roughly 130 private and public schools, entering grades 7-12 in the 1994-1995 school year. The large sample size allows to conduct empirical investigations on black and white pupils. The richness of information on family background, school quality and area of residence is essential in our context, since pupils’ percentage of same-race friends may proxy for unobservable family, school or neighborhood characteristics.\(^9\) AddHealth contains unique information on friendship association in the school. It is based on actual friends nominations. All students were asked to list their best friends (up to five for each sex), and it is possible to assemble the characteristics for each friend. We exploit this feature of the dataset to derive the share of same-race aim at controlling for school and neighborhood segregation effects. In other words, we want to exclude the possibility that the estimated coefficient on our target variable, i.e. percentage of same-race friends, only reflects segregation effects.

\(^8\)More precisely, the impact of say, \( \tilde{x}_{ij} \) on \( y_{ij} \) is \[ \frac{\partial y_{ij}}{\partial x_{ij}} = \alpha_{1j} + \alpha_{4j} k_{ij}. \] It will be evaluated at the sample average of \( k_{ij} \).

friends each student actually nominates as best friends. This will be the empirical counterpart of $\tilde{x}_{ij}$, for both blacks and whites.

Concerning $y_{ij}$, the individual’s school performance, we use the available information on the grade achieved by each student in mathematics, history and social studies and science. On the basis of these scores and adopting the standard approach in the sociological literature, a school performance index is calculated for each respondent. The mean is 2.45 and the standard deviation is equal to 2.23. The value of the Cronbach-$\alpha$, which is equal to 0.84 ($0 \leq \alpha \leq 1$), points to the good quality of the derived indicator. The index is then normalized to be between 0 and 1.

The other key variable in the theoretical model is parent’s education, $k_{ij}$ and $\overline{k}_{ij}$. The AddHealth contain information about the schooling of the (biological or non-biological) parent who is living with the child, distinguishing between “never went to school”, “not graduate from high school”, “high school graduate”, “graduated from college or a university”, “professional training beyond a four-year college”. We use this information, considering only the education of the father if both parents are in the household.

Table 1 reports descriptive statistics of our key variables. Differences of mean values between races are always statistically significant. It appears that whites are less open towards interracial contacts than blacks. The average white student has almost 85 percent of white friends whereas the average black has less than 60 percent black friends (with considerable variations around this average value). Looking at the inter-race differences in academic performance and parental education, not surprisingly, blacks perform on average worse at school than whites and have lower educated parents.

\[\text{Insert Table 1 here}\]

\(^{10}\)Clearly, we do not consider the absolute number of same-race friends but the percentage in total number of nominations to avoid problems arising from the presence of unobserved factors (such as wearing particular clothes or having special electronic gadgets) that might induce a teenager to be more popular among her/his same-race (or different race) peers regardless ethnic preferences.

\(^{11}\)They range from D or lower to A, the highest grade, and are re-coded 1 to 4.

\(^{12}\)The analysis presented in this paper has also been performed separately on each discipline. The results on our target variables remain qualitatively roughly unchanged.

\(^{13}\)The schooling of the parent is coded as 1=never went to school, 2= not graduate from high school, 3= high school graduate, 4=graduated from college or a university, 5= professional training beyond a four-year college. We also performed an alternative analysis splitting our sample of students in five groups according to the education level of the parents, and then estimating each model for each sub-sample separately. The results remain qualitatively unchanged.
4 Empirical results: Exogenous choice of friends

Table 2 reports the estimation results of Model 1 on our target coefficients, $\alpha_1 j$, $\alpha_2 j$, $\alpha_3 j$ and $\alpha_4 j$ for both blacks and whites. Their estimated coefficients are all statistically significant for both whites and blacks, providing evidence of the importance of ethnic preferences (in choosing friends) and parental education in shaping students’ academic performance. Evaluating the impacts of $x_{ij}$ and $k_{ij}$ on $y_{ij}$ at the sample averages of the variables for the two groups, we find that their signs are exactly the ones predicted by the theoretical model as exposed in (i) and (ii) in Proposition 1. In particular, $\frac{\partial y_{iB}^*}{\partial x_{iB}} = \alpha_1 B + \alpha_4 B k_{iB} < 0$ and $\frac{\partial y_{iW}^*}{\partial x_{iW}} = \alpha_1 W + \alpha_4 W k_{iW} > 0$, which means that for blacks, the higher their percentage of black friends, the lower their schooling performance while we have exactly the reverse for white pupils. Observe that this implies that the inter-race difference in terms of schooling achievement is an increasing function of ethnic identity (as measure by the percentage of same-race friends). In other words, looking across races, holding all the control variables constant, the more individuals conform to their group norm, the larger are the inter-race differences in terms of academic performance. Interestingly, looking at the magnitude of the estimated coefficients, in terms of standard deviations, the effect (in absolute value) of same race friends on school performance is similar for blacks and whites, with the impact being slightly more potent in the former group. Indeed, raising the percentage of black friends of a black pupil by a standard deviation would decrease his/her academic performance by roughly 14 percent of a standard deviation, whereas a standard deviation increase in the percentage of white friends of a white pupil would increase his/her academic performance by roughly 11 percent. Regarding parental education, the effect in standard deviations is instead roughly double for blacks than for whites. A standard deviation increase in parental education for a black guy would produce some 40 percent of a standard deviation increase in school performance, while this effect drops to roughly 20 percent for whites.

[Insert Table 2 here]

Let us now investigate the cross effects. We are mainly interested in $\alpha_4 j = \frac{\partial^2 y_{ij}^*}{\partial x_{ij} \partial k_{ij}}$, $j = B, W$.

Table 2 also shows that this estimated coefficient is statistically significant for both whites and blacks. They also have the predicted signs showed in results (iii) and (iv) in Proposition 1, that is $\alpha_4 B < 0$ and $\alpha_4 W > 0$. This shows that the impact of a pupil’s ethnic identity on schooling is in fact influenced by the teenager’s parental education. Specifically, we find that, for whites, the effect of same-race friends on schooling is an increasing function of parental education, i.e. the more the parents are educated, the higher the influence of ethnic identity on schooling. On the contrary, for blacks, the negative effect of ethnic identity on schooling is a decreasing function of parental

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The qualitative results on our target variables are robust to alternative sets of control variables.
education, i.e. the more the parents are educated, the lower (in absolute value) is the influence of same-race friends on schooling. One way to interpret this result in light of the theoretical model is that the attachment of black children to their culture of origin is higher the less educated the parents.

All these results provide evidence on the importance of ethnic preferences in explaining the inter-race achievement gap. Of course, as already discussed above, the choice of same-race friends is an endogenous variable and should be dealt with adequately. This is what is done in the next section.

5 Empirical results: Endogenous choice of friends

We would like now to test Propositions 3 and 4.

To find appropriate instruments to identify the preferences equations expressed by (5) is a difficult task. In the theoretical model, the key exogenous mechanism driving the choice of the race of a (black) student’s friends is the individual’s attachment to his/her culture of origin, referred as the sense of black identity, $z_{iB}$. We thus would like to measure ethnic identity. We exploit the detailed information provided in our data on the ethnicity of the parents (interracial marriage)\(^{15}\) and on various aspects of religious participation of the adolescent, i.e. religious affiliation, importance of religion, religious service attendance, involvement in church youth groups and fanatical faith attitude (e.g. thinking of being born-again-Christian). Appendix 1 gives a precise definition of these instrumental variables. Table 3 provides their descriptive statistics in our sample.

The choice of the instruments is grounded on the following reasons.

First, interracial marriage is typically considered as a sign of inclination toward cultural assimilation, especially for African Americans (see, in particular, Aldridge, 1978; Qian, 1999; Meng and Gregory, 2005; Tucker and Mitchell-Kernan, 1990; Zebroski, 1999). Indeed, children of parents with homogamy marriage are in general less exposed to interracial contacts, thus having a higher percentage of same race friends. The ethnicity of the parents should not, however, directly influence the education attainment of the children, nor it is caused by this variable. Interestingly, in our data set, nearly 20 percent of black pupils have one of his/her parent who is not black (see Table 3).

Second, regarding the other set of instruments, racial differences in religious affiliation and participation in the United States are well-documented (Gallup and Castelli, 1989). It is also well-established that religion activities have an important impact of black’s sense of identity. Indeed,

\(^{15}\)Black adolescents with parents of different race are adolescents who define themselves as Black/African Americans but have either their father or their mother who is non Black/African American.
the black church is the anchoring institution in the African American community (Lincoln and Mamiya, 1990; Myrdal, 1944). The church acts simultaneously as a school, a benevolent society, a political organization, a spiritual base, etc. As one of the few institutions owned and operated by African Americans, the church is often the center of activity in black community. In particular, the black church has a documented tradition of involvement in extra religious civic and political activities (Findlay, 1993; Harris, 1994). Black churches are significantly more likely than white congregations to participate in civil rights activities. Lincoln and Mamiya (1990) find that 90 percent of the clergy approved of their clerical peers’ taking part in protest marches on civil right issues. Using the data from the 1979-1980 national Survey of Black Americans, Ellison (1993) show that participation in church communities fosters positive self-perception of blackness through the interpersonal supportiveness and positive reflected appraisals of coreligionsists. Using data from more than three years of ethnographic research in Groveland, an African American neighborhood in Chicago, Pattillo-McCoy (1999) founds that the black church provides a cultural blueprint for civic life in the neighborhood and shows the power of church rituals as cultural tools for facilitating local organizing and invigorating activism among African American.

Using the same data base employed in this paper (Add Health data), Smith et al. (2002) provide a detailed picture of the religious lives of adolescent in the US. Among their finding, they show that African-American adolescents have the highest rates of church attendance, and they tend to cluster in specific religious groups (e.g. African methodist, Holiness, Islam, Jehovah’s witness, Baptist). In addition, the followers of some of these traditions (e.g. African methodist, Holiness) are for the large majority blacks. Table 3 documents that almost 40% of the black students in our sample are African methodists or follow the Islam or Holiness religions. Therefore, variations in the religious behavior across black teenagers might be related to variations in the percentage of same-race friends, given that, e.g., a high participation in religious youth groups certainly increase the sense of black identity and the probability to meet same-race fellows if the religious tradition is one of those mainly followed by blacks. However, religious affiliation or participation should not influence the student’s school performance nor they are determined by this variable. In other words, it is reasonable to think that the chosen instruments do not affect the teenagers’ school performance other than through peers effects stemming from the choice of same-race friends.

Thus, our empirical analysis proceeds as follows. In the first stage, we model the percentage of black friends as a function of a set of traditional variables that might affect this choice and the instruments capturing the individual attachment to the race-group, i.e. the sense of black identity, $z_{iB}$, in the theoretical model. The predicted levels of percentage of same-race black friends are then used in the second stage model (Model 1), which includes all the controls of the first step model with the exclusion of the variables acting as instruments. Using a likelihood ratio test, we find that we cannot reject the hypothesis that the chosen exclusion restrictions are valid. This offers a statistical support to the validity of the chosen instruments.
5.1 First stage: What determines the choice of same-race friends?

Table 4 presents the results from the first-stage estimation. All the estimated coefficients of the instruments, in terms of both ethnicity of the parents and religious activities, behave as expected and are statistically significant at all conventional levels of significance. For example, having a parent of a different race leads black teenagers to assimilate more to the white culture by choosing more white friends, and the effect is large in magnitude (the share of black friends decreases by roughly 25 percent). On the contrary, any religious activity tends to strengthen the sense of black identity by inducing black teenagers to choose more black best friends. In particular, being a disciple of African methodist, Holiness or Islam religions increases the percentage of black friends by almost 40 percent. All of this is consistent with the results in the sociological literature mentioned above that show that participation in religious activities and homogamy marriage are a clear indication of black identity.

5.2 Second stage: What are the effects on education?

The results from our instrumental variable education regression are given in Table 5 (second stage). As stated above, the willingness of blacks to interact with blacks captures to some extent the black identity. The question then is whether there is a negative externality from not associating with the majority group in term of education outcome, here measured as test scores. This is indeed what is found since there is a penalty of choosing black friends. Increasing the percentage of black friends of a black pupil by a standard deviation would decrease his/her academic performance by roughly 13 percent of a standard deviation. Interestingly, the results remain qualitatively unchanged with respect to those presented in Table 2 for the non-instrumental variables estimates. Thus, having an identity that is closely tied to one’s ethnic group does generate an education penalty.

6 Conclusion

Friendships are complex social relationships. The study of interethnic and interracial interactions and relationships among youth, also called intergroup relations, has become a critical, complex, and challenging field in recent years, especially in sociology. This is obviously a crucial question since it deals with identity. There is a consensus in psychology and sociology that the first influences on
a child’s identity formation —how the child comes to see himself or herself as a member of one or several racial, ethnic, cultural, and religious groups— occurs at home and in the context of family. Most significant at the earliest stages are parents and their values, specifically how they deal with issues of race, in deeds and in words. Also important is the community in which the child lives, and the messages the child encounters; later on, peers, teachers, school officials, and community leaders have significant influence.

In the present paper, we endeavour to analyze the impact of youth racial friendships on school performance, accounting for the influence of parental inputs. For that, we develop and test a model where black and white adolescents choose the percentage of same-race friends as well as their education effort. We find that choosing black friends has a negative impact on blacks’ test scores while choosing white friends has a positive impact on both blacks’ and whites’ test scores. The negative effect of black friends is mitigated by parents’ education while the positive effect of white friends is amplified by parents’ education. Though one needs to be cautious in this type of analysis, our results do reveal that there is an education penalty associated with such extreme identities. These effects are evident when we control for the endogeneity of ethnic preferences (black identity).

Interestingly, using data from the National Educational Longitudinal Study, Cheng and Starks (2002) found that African American parents tend to hold higher educational aspirations for their children than do white parents, but the relative influence of African American fathers on students’ education expectation are smaller than those of their white counterparts. Also, with the same data set, Ainsworth-Darnell and Downey (1998) found that African-American friendship groups were more proschool and more admiring of academic achievers than were friendship groups of other races. These findings may relate to those of Clark and Ayers (1988), who found that whites’ friendships were matched more closely on academic achievement, while African Americans’ friendships were matched more closely on achievement-oriented aspects of personality. Clearly, this issue is controversial and it is difficult to draw clear conclusion. In the present research, even if we could not disentangle all the elements of identity and its impact on education, we could however sort out some of its thicker threads.

References


Appendix 1: Proofs of Propositions of the Theoretical Model

**Proof of Proposition 1:** The program of each individual $ij$ is:

$$
\max_{y_{ij}} \left\{ k_{ij} y_{ij}^\beta \left[ x_{ij}k_B + (1 - x_{ij})k_W \right] - \frac{1}{2} y_{ij}^2 \right\}
$$

The first order condition of this problem gives:

$$
\frac{\partial U_{ij}(y_{ij})}{\partial y_{ij}} = \beta_1 k_{ij} y_{ij}^{\beta_1 - 1} k_W - y_{ij} = 0
$$

(14)

where $k_W$ is defined by (2). The second order condition is determined by:

$$
\frac{\partial^2 U_{ij}(y_{ij})}{\partial y_{ij}^2} = (\beta_1 - 1) \beta_1 k_{ij} y_{ij}^{\beta_1 - 2} k_W - 1 < 0
$$

and is always satisfied since $\beta_1 < 1$.

Let us now analyze the optimal choice of individual $ij$. Using (14), we have:

$$
y_{ij}^* = \beta_1^{1/(2-\beta_1)} \left( k_W k_W \right)^{1/(2-\beta_1)}
$$

(15)

Let us now focus on whites. Equation (15) can be written as:

$$
y_{iW}^* = \beta_1^{1/(2-\beta_1)} k_{iW}^{1/(2-\beta_1)} k_W^{1/(2-\beta_1)}$$

$$
= \beta_1^{1/(2-\beta_1)} k_{iW}^{1/(2-\beta_1)} \left[ x_{iW} k_B + (1 - x_{iW}) k_W \right]^{1/(2-\beta_1)}$$

$$
= \beta_1^{1/(2-\beta_1)} k_{iW}^{1/(2-\beta_1)} \left[ (1 - x_{iW}) k_B + x_{iW} k_W \right] \left[ (1 - x_{iW}) k_B + x_{iW} k_W \right]^{1/(2-\beta_1)}
$$

(16)

where $x_{iW} = 1 - x_{iW}$ is the number of white friends of individual $ij$. Thus

$$
\frac{\partial y_{iW}^*}{\partial x_{iW}} = \beta_1^{1/(2-\beta_1)} \left( k_{iW}^{(1-1)/(2-\beta_1)} k_W^{(1-1)/(2-\beta_1)} - k_B \right) > 0
$$

This is positive because of (H1) and the fact that $\beta_1 < 1$. Moreover, we have:

$$
\frac{\partial^2 y_{iW}^*}{\partial x_{iW} \partial k_{iW}} = \beta_1^{1/(2-\beta_1)} \left( k_{iW}^{(1-1)/(2-\beta_1)} k_W^{(1-1)/(2-\beta_1)} - k_B \right) > 0
$$

Let us now focus on blacks. Equation (15) can be written as:

$$
y_{iB}^* = \beta_1^{1/(2-\beta_1)} k_{iB}^{1/(2-\beta_1)} k_B^{1/(2-\beta_1)}$$

$$
= \beta_1^{1/(2-\beta_1)} k_{iB}^{1/(2-\beta_1)} \left[ x_{iB} k_B + (1 - x_{iB}) k_W \right]^{1/(2-\beta_1)}
$$

(17)

By differentiating this equation, we obtain:

$$
\frac{\partial y_{iB}^*}{\partial x_{iB}} = -\beta_1^{1/(2-\beta_1)} k_{iB}^{1/(2-\beta_1)} \left( k_B - k_W \right) < 0
$$
This is negative because of \((H1)\) and the fact that \(\beta_1 < 1\). Moreover, we have:

\[
\frac{\partial^2 y_{iB}}{\partial x_{iB} \partial k_{iB}} = -\frac{\beta_1^{1/(2-\beta_1)}}{(2-\beta_1)^2 k_{iB}} \frac{1}{(2-\beta_1)\beta_1^{1/(2-\beta_1)}} k_{iB}^{\beta_1-1)/(2-\beta_1)} (k_{W} - k_{B}) < 0
\]

\[
\text{Proof of Proposition 2:}\] Let us solve the first stage of this model. Each teenager \(ij\) solves the following program respectively:

\[
\max_{\tilde{x}_{ij}} \left\{ z_{ij} \tilde{x}_{ij} - \frac{1}{2} b \tilde{x}_{ij}^2 + d \beta_1^{1/(2-\beta_1)} k_{ij}^{1/(2-\beta_1)} \frac{1}{k_{ij}} \frac{1}{(2-\beta_1)\beta_1^{1/(2-\beta_1)}} (k_{W} - k_{B}) \right\}
\]

where \(\tilde{x}_{ij}\) is the percentage of same-race friends, that is: \(\tilde{x}_{iB} = x_{iB}\) and \(\tilde{x}_{iW} = 1 - x_{iW}\) and \(k_{ij}\) is determined by \((2)\) and given by:

\[
k_{iB} = \tilde{x}_{iB} \overline{k}_{B} + (1 - \tilde{x}_{iB}) \overline{k}_{W}
\]

for blacks and

\[
k_{iW} = (1 - \tilde{x}_{iW}) \overline{k}_{B} + \tilde{x}_{iW} \overline{k}_{W}
\]

for whites. We have:

\[
\frac{\partial \overline{k}_{iB}}{\partial \tilde{x}_{iB}} = - (\overline{k}_{W} - \overline{k}_{B}) < 0
\]

\[
\frac{\partial \overline{k}_{iW}}{\partial \tilde{x}_{iW}} = \overline{k}_{W} - \overline{k}_{B} > 0
\]

Let us now solve the first-order condition:

\[
\frac{\partial V_{ij}(x_{ij})}{\partial \tilde{x}_{ij}} = z_{ij} - b \tilde{x}_{ij} + 1_j d \beta_1^{1/(2-\beta_1)} k_{ij}^{1/(2-\beta_1)} (k_{W} - k_{B})
\]

where \(1_B = -1\) and \(1_W = 1\). The second order conditions (SOCs) is given by:

\[
\frac{\partial^2 V_{ij}(x_{iB})}{\partial \tilde{x}_{ij}^2} = -b - 1_j d (\beta_1 - 1) \beta_1^{1/(2-\beta_1)} k_{ij}^{1/(2-\beta_1)} (k_{W} - k_{B})^2 < 0
\]

For blacks, the SOC is always satisfied because \(0 < \beta_1 < 1\). For whites, we assume that \(b\) is large enough for the SOC to be satisfied, that is:

\[
b > -d (\beta_1 - 1) \beta_1^{1/(2-\beta_1)} \frac{1}{(2-\beta_1)^2} k_{ij}^{1/(2-\beta_1)} (k_{W} - k_{B})^2
\]

Let us solve the problem of an adolescent \(ij\). Solving \((21)\) and using \((19)\) and \((20)\) lead to the following first order condition:

\[
z_{ij} - b \tilde{x}_{ij} = -1_j d (2-\beta_1) \beta_1^{1/(2-\beta_1)} k_{ij}^{1/(2-\beta_1)} (k_{W} - k_{B})
\]
Let us now solve the problem of a black adolescent. The first order condition is given by (23). Let us have the following notations:

\[ f_B(x_{iB}) = \frac{z_{iB} - b x_{iB}}{K_B} \]  

where \( K_B \) is defined by:

\[ K_B \equiv d^{\frac{1}{(2-\beta_1)}} \frac{\beta_1^{1/(2-\beta_1)}}{2 - \beta_1} k_{iB}^{\frac{1}{(2-\beta_1)}} (K_W - \overline{K}_B) > 0 \]  

with

\[ \frac{\partial K_B}{\partial k_{iB}} = K_B'(k_{iB}) = d^{\frac{1}{(2-\beta_1)}} \frac{\beta_1^{1/(2-\beta_1)}}{2 - \beta_1} k_B^{\frac{1}{(2-\beta_1)}} (K_W - \overline{K}_B) > 0 \]  

and where

\[ g_B(x_{iB}) \equiv k^{\frac{(\beta_1-1)/(2-\beta_1)}}{iB} \]

Thus, solving (23) is like solving the following equation:

\[ f_B(x_{iB}) = g_B(x_{iB}) \]  

We have:

\[ f_B(0) = \frac{z_{iB}}{K} > 0 \ , \ f_B(1) = \frac{z_{iB} - b}{K} \]

\[ f'_B(x_{iB}) = -\frac{b}{K} < 0 \]

\[ g_B(0) = k^{\frac{(\beta_1-1)/(2-\beta_1)}}{W} > 0 \ , \ g_B(1) = k^{\frac{(\beta_1-1)/(2-\beta_1)}}{B} > 0 \]

with \( g_B(0) < g_B(1) \) since \( (\beta_1 - 1) / (2 - \beta_1) < 0 \). Also,

\[ g'_B(x_{iB}) = -\frac{(\beta_1 - 1)}{2 - \beta_1} k_B^{\frac{2(\beta_1-3)/(2-\beta_1)}} (K_W - \overline{K}_B) > 0 \]

Furthermore, we have:

\[ g''_B(x_{iB}) = -\frac{(\beta_1 - 1)}{2 - \beta_1} \left( \frac{2(\beta_1-3)/(2-\beta_1)}{2 - \beta_1} \right) k_B^{\frac{2(\beta_1-3)/(2-\beta_1)}} (K_W - \overline{K}_B)^2 > 0 \]

We have the following straightforward result (observe that \( f_B(1) < 0 \) when \( \frac{1}{2} < \frac{z_{iB}}{b} < 1 \) and \( f_B(1) \geq 0 \) when \( \frac{z_{iB}}{b} \geq 1 \)):

\[ \text{Lemma 1} \quad \text{Assume that (H1) holds. For black adolescents, we have:} \]

(i) Assume that

\[ \frac{1}{2} < \frac{z_{iB}}{b} < 1 \]

(i1) if \( g_B(0) > f_B(0) \), then \( x_{iB}^* = 0 \).
\( \text{(i)} \) if \( g_B(0) \leq f_B(0) \leq g_B(1) \), then \( 0 < x^*_{iB} < 1 \);
\( \text{(ii)} \) Assume that \( \frac{z_{iB}}{b} \geq 1 \)
\( \text{(ii1)} \) if \( g_B(0) > f_B(0) \), then \( x^*_{iB} = 0 \).
\( \text{(ii2)} \) if \( f_B(1) \leq g_B(0) \leq f_B(0) \leq g_B(1) \), then \( 0 < x^*_{iB} < 1 \);
\( \text{(ii3)} \) if \( f_B(1) \leq g_B(0) \leq g_B(1) \leq f_B(0) \), then \( 0 < x^*_{iB} < 1 \);
\( \text{(ii4)} \) if \( f_B(0) \leq f_B(1) \leq f_B(0) \leq g_B(1) \), then \( 0 < x^*_{iB} < 1 \);
\( \text{(ii5)} \) if \( g_B(0) \leq f_B(1) \leq g_B(1) \leq f_B(0) \), then \( 0 < x^*_{iB} < 1 \);
\( \text{(ii6)} \) if \( g_B(1) < f_B(1) \), then \( x^*_{iB} = 1 \).

Proposition 2 is the equivalent of this Lemma using the values in (29) and (30).

Let us now characterize the case for which \( x^*_{iB} = 1 \) (i.e. when (9) holds). Using (3), we have:
\[
y^*_{iB} = \beta_1^{1/(2-\beta_1)} k_{iB}^{1/(2-\beta_1)} K_B^{-1/(2-\beta_1)}
\]
and it is easy to see that
\[
\frac{\partial y^*_{iB}}{\partial k_{iB}} > 0, \quad \frac{\partial y^*_{iB}}{\partial k_B} > 0
\]
Let us then characterize the case for which \( x^*_{iB} = 0 \) (i.e. when (7) holds). Using (3), we have:
\[
y^*_{iB} = \beta_1^{1/(2-\beta_1)} k_{iB}^{1/(2-\beta_1)} K_W^{-1/(2-\beta_1)}
\]
and it is easy to see that
\[
\frac{\partial y^*_{iB}}{\partial k_{iB}} > 0, \quad \frac{\partial y^*_{iB}}{\partial k_W} > 0
\]

\[\blacksquare\]

**Proof of Proposition 3:** Let us now solve the choice of a white teenager. The first order condition is given by (23) that we rewrite as:
\[
\frac{\partial V_{iW}(\tilde{x}_{iW})}{\partial \tilde{x}_{iW}} = d \beta_1^{1/(2-\beta_1)} k_{iW}^{1/(2-\beta_1)} K_W^{-1/(2-\beta_1)} (K_W - K_B) - (b \tilde{x}_{iW} - z_{iW}) = 0
\]
where
\[
K_{iW} = (1 - \tilde{x}_{iW}) K_B + \tilde{x}_{iW} K_W \tag{32}
\]
We have the following notations:
\[
f_W(\tilde{x}_{iW}) = \frac{b \tilde{x}_{iW} - z_{iW}}{K_W} \tag{33}
\]

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where $K_{W}$ is defined by:

$$K_{W} \equiv \frac{d^{1/(2-\beta_{1})}}{(2-\beta_{1})} k_{iW}^{1/(2-\beta_{1})} (\bar{K}_{W} - \bar{K}_{B}) > 0 $$

(34)

with

$$\frac{\partial K_{W}}{\partial k_{iW}} = K'(k_{iW}) = \frac{d^{1/(2-\beta_{1})}}{(2-\beta_{1})} k_{iW}^{(\beta_{1}-1)/(2-\beta_{1})} (\bar{K}_{W} - \bar{K}_{B}) > 0 $$

(35)

and

$$g_{W}(\bar{x}_{iW}) = \bar{k}_{iW}^{(\beta_{1}-1)/(2-\beta_{1})} $$

(36)

Thus, solving (31) is like solving the following equation:

$$f_{W}(\bar{x}_{iW}) = g_{W}(\bar{x}_{iW}) $$

(37)

We have:

$$f_{W}(0) = -\frac{z_{iW}}{K_{W}} < 0 , f_{W}(1) = \frac{b - z_{iW}}{K_{W}} $$

(38)

$$f'_{W}(\bar{x}_{iW}) = \frac{b}{K_{W}} > 0 $$

(39)

$$g_{W}(0) \equiv \bar{k}_{iW}^{(\beta_{1}-1)/(2-\beta_{1})} > 0 , g_{W}(1) = \bar{k}_{W}^{(\beta_{1}-1)/(2-\beta_{1})} $$

(40)

with $g_{W}(0) > g_{W}(1)$ since $(\beta_{1} - 1) / (2 - \beta_{1}) < 0$ and $\bar{k}_{B} < \bar{k}_{W}$.

Furthermore,

$$g''_{W}(\bar{x}_{iW}) = \frac{(\beta_{1} - 1)(2\beta_{1} - 3)}{(2 - \beta_{1})^{2}} k_{iW}^{(3\beta_{1}-3)/(2-\beta_{1})} (\bar{K}_{W} - \bar{K}_{B})^{2} > 0 $$

We have the following straightforward result (observe that $f_{W}(0) < 0$ when $\frac{1}{2} < \frac{z_{iW}}{b} < \frac{1}{2}$ and $f_{W}(0) \geq 0$ when $\frac{z_{iW}}{b} \geq 1$):

Lemma 2 Assume that (H1) holds. Then, for white adolescents, we have:

(i) Assume that:

$$\frac{1}{2} < \frac{z_{iW}}{b} < 1 $$

(i1) if $g_{W}(1) > f_{W}(1)$, then $\bar{x}_{iW} = 1$.

(i2) if $g_{W}(1) \leq f_{W}(1) \leq g_{W}(0)$, then $0 < \bar{x}_{iW}^{(2)} < 1$, with $\bar{x}_{iW}^{(2)} > \max \{1/2, z_{iW}/b\}$.

(i3) if $g_{W}(0) < f_{W}(1)$, then $0 < \bar{x}_{iW}^{(3)} < 1$, with $\bar{x}_{iW}^{(3)} > \bar{x}_{iW}^{(2)} > \max \{1/2, z_{iW}/b\}$.
(ii) Assume that
\[ \frac{z_{iW}}{b} \geq 1 \]
then \( \tilde{x}_{iW}^* = 1 \).

**Proof.** Observe first that when \( z_{iW} \geq b \geq b\tilde{x}_{iW} \), \( \forall \tilde{x}_{iW} \in [0,1] \), then \( \frac{\partial V_{iW}(\tilde{x}_{iW})}{\partial \tilde{x}_{iW}} > 0 \) and thus \( \tilde{x}_{iW}^* = 1 \). This is case (ii). Now, when \( \frac{1}{2} < \frac{z_{iW}}{b} < 1 \), two cases may arise. First, for \( 0 < \tilde{x}_{iW} \leq 1/2 \), \( z_{iW} - b\tilde{x}_{iW} \geq 0 \) and thus \( \tilde{x}_{iW}^* = 1 \). As a result, it has to be that \( \tilde{x}_{iW} > 1/2 \) to have an interior solution. Now, for \( \tilde{x}_{iW} > 1/2 \), \( z_{iW} - b\tilde{x}_{iW} \leq 0 \). In order to have an interior solution, it has to be that \( z_{iW} - b\tilde{x}_{iW} < 0 \), which is equivalent to \( \tilde{x}_{iW} > z_{iW}/b \). As a result, there will be an interior solution if and only if \( \tilde{x}_{iW} > \max \{1/2, z_{iW}/b\} \). Now solving the different values of \( f_W(.) \) and \( g_W(.) \) gives the results in cases (i1), (i2) and (i3).

Finally, Proposition 3 is the equivalent of this Lemma using the values in (38) and (39).

**Proof of Proposition 4:** Let us characterize the interior solution for blacks for which \( 0 < x_{iB}^* < 1 \) (i.e. when both (9) and (7) do not hold). Figure 3a illustrates an interior equilibrium for case (ii) in Lemma 1, that is when \( g_B(0) \leq f_B(1) \leq g_B(1) \leq f_B(0) \). In the case of an interior solution, \( x_{iB}^* \) is implicitly defined by (23) or equivalently by:
\[ f_B(x_{iB}) - g_B(x_{iB}) = 0 \] (41)
where \( f_B(x_{iB}) \) and \( g_B(x_{iB}) \) are respectively defined by (24) and (27). We can perform some comparative statics exercise. By totally differentiating (23) and using the implicit function theorem, we obtain:
\[ \frac{\partial x_{iB}^*}{\partial z_{iB}} = \frac{1}{K(k_{iB})} \left[ f'_B(x_{iB}) - g'_B(x_{iB}) \right] \geq 0 \] (42)
\[ \frac{\partial x_{iB}^*}{\partial k_{iB}} = \frac{(z_{iB} - b x_{iB}^*) K'(k_{iB})}{[K(k_{iB})]^2} \left( f'_B(x_{iB}) - g'_B(x_{iB}) \right) \leq 0 \] (43)
\[ \frac{\partial x_{iB}^*}{\partial d} = \frac{\beta_1}{2 - \beta_1} \frac{k_{iB}^{1/2} - k_{iB}^{1/2} - 1}{(2 - \beta_1) (2 - 1/2)} \frac{f'_B(x_{iB}) - g'_B(x_{iB})}{k_{iB}^{1/2} - k_{iB}^{1/2}} < 0 \] (44)
where
\[ f'_B(x_{iB}) - g'_B(x_{iB}) = -k_{iB}^{1/2} + \frac{(\beta_1 - 1)}{2 - \beta_1} \frac{k_{iB}^{3/2} - k_{iB}^{1/2}}{k_{iB}^{3/2} - k_{iB}^{1/2}} (k_{iB}^{3/2} - k_{iB}^{1/2}) < 0 \]
and \( K'_B(k_{iB}) > 0 \) is given by (26). By further differentiating either (42) or (43), we obtain:
\[ \frac{\partial^2 x_{iB}}{\partial z_{iB} \partial k_{iB}} = \frac{K'_B(k_{iB}) \left[ f'_B(x_{iB}) - g'_B(x_{iB}) \right] + K_B(k_{iB}) \left[ \frac{\partial f_B(x_{iB})}{\partial k_{iB}} - \frac{\partial g_B(x_{iB})}{\partial k_{iB}} \right]}{\left( K_B(k_{iB}) \left[ f_B(x_{iB}) - g_B(x_{iB}) \right] \right)^2} \]
where
\[ \frac{\partial f'_B(x_{iB})}{\partial k_{iB}} - \frac{\partial g'_B(x_{iB})}{\partial k_{iB}} = b K'_B(k_{iB}) \left[ K_{iB}(k_{iB}) \right]^{-2} \left( \frac{(\beta_1 - 1)}{2 - \beta_1} \frac{k_{iB}^{3/2} - k_{iB}^{1/2}}{k_{iB}^{3/2} - k_{iB}^{1/2}} (k_{iB}^{3/2} - k_{iB}^{1/2}) \right) \frac{\partial x_{iB}^*}{\partial k_{iB}} > 0 \]

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Thus the sign of $\frac{\partial^2 x_B}{\partial z_B \partial k_i B}$ is the same as the sign of

$$K' B(k_i B) \left[ f'_B(x_i B) - g'_B(x B) \right] + K(k_i B) \left[ \frac{\partial f_B(x_i B)}{\partial k_i B} - \frac{\partial g'_B(x_i B)}{\partial k_i B} \right]$$

$$= K' B(k_i B) \left[ f'_B(x_i B) - g'_B(x B) \right] + K B(k_i B) \left[ \frac{b - (2\beta_i - 3) (\beta_i - 1) (3\beta_i - 5)/(2 - \beta_i) (z_i B - b x_i B) (\mathcal{F}_W - \mathcal{F}_B)}{f'_B(x_i B) - g'_B(x B)} \right]$$

$$= K' B(k_i B) \left\{ f'_B(x_i B) - g'_B(x B) + \frac{b}{K B(k_i B)} \frac{(2\beta_i - 3) (\beta_i - 1) (3\beta_i - 5)/(2 - \beta_i) (z_i B - b x_i B) (\mathcal{F}_W - \mathcal{F}_B)}{(2 - \beta_i)^2} \right\}$$

Since $K' B(k_i B) > 0$, we are now looking at the sign of

$$f'_B(x_i B) - g'_B(x B) + \frac{b}{K B(k_i B)} \frac{(2\beta_i - 3) (\beta_i - 1) (3\beta_i - 5)/(2 - \beta_i) (z_i B - b x_i B) (\mathcal{F}_W - \mathcal{F}_B)}{(2 - \beta_i)^2}$$

$$= \frac{(\beta_i - 1)}{(2 - \beta_i)^3} (2\beta_i - 3)/(2 - \beta_i) (\mathcal{F}_W - \mathcal{F}_B) - \frac{(2\beta_i - 3) (\beta_i - 1) (z_i B - b x_i B) (\mathcal{F}_W - \mathcal{F}_B)}{(2 - \beta_i)^2}$$

So now the sign of $\frac{\partial^2 x_B}{\partial z_B \partial k_i B}$ is the same as the sign of

$$\frac{(2\beta_i - 3) (z_i B - b x_i B) (\mathcal{F}_W - \mathcal{F}_B)}{d\beta_i/(2 - \beta_i)} - (\mathcal{F}_W - \mathcal{F}_B)$$

We would like to show that

$$\frac{\partial^2 x_B}{\partial z_B \partial k_i B} < 0 \Leftrightarrow \frac{(2\beta_i - 3) (z_i B - b x_i B) (\mathcal{F}_W - \mathcal{F}_B)}{d\beta_i/(2 - \beta_i)} < (\mathcal{F}_W - \mathcal{F}_B)$$

$$\frac{b (2 - \beta_i)}{k_i B(2 - \beta_i)} + \frac{(1 - \beta_i) d\beta_i/(2 - \beta_i)}{(2 - \beta_i)} (\mathcal{F}_W - \mathcal{F}_B)^2 > (3 - 2\beta_i) (z_i B - b x_i B) (\mathcal{F}_W - \mathcal{F}_B)^2$$

This is true if:

$$\frac{(1 - \beta_i) d\beta_i/(2 - \beta_i)}{(2 - \beta_i)} (\mathcal{F}_W - \mathcal{F}_B)^2 > (3 - 2\beta_i) (z_i B - b x_i B) (\mathcal{F}_W - \mathcal{F}_B)^2$$
\( \Leftrightarrow (1 - \beta_1) d\beta_1^{1/(2-\beta_1)}k_{iB}^{\beta_1/(2-\beta_1)} (\bar{k}_W - \bar{k}_B)^2 > (2 - \beta_1)(3 - 2\beta_1)(z_iB - b x_i^B) \)

\( \Leftrightarrow (1 - \beta_1) d\beta_1^{1/(2-\beta_1)}k_{iB}^{\beta_1/(2-\beta_1)} (\bar{k}_W - \bar{k}_B)^2 + b x_i^B (2 - \beta_1)(3 - 2\beta_1) > (2 - \beta_1)(3 - 2\beta_1)z_iB \)

This is true if:

\[
 z_iB < d (1 - \beta_1) \beta_1^{1/(2-\beta_1)}k_{iB}^{\beta_1/(2-\beta_1)} (\bar{k}_W - \bar{k}_B)^2
\]

(45)

Since \( k_{iB} \) is decreasing in \( z_iB \) (indeed \( \frac{\partial k_{iB}}{\partial z_iB} = - (\bar{k}_W - \bar{k}_B) \frac{\partial x_i^B}{\partial z_iB} < 0 \)), then this condition is true if \( z_iB \) is low enough or \( d \) is large enough.

Using (3), we can now calculate \( y_i^* \), which is given by:

\[
y_i^* = \beta_1^{1/(2-\beta_1)}k_{iB}^{1/(2-\beta_1)}(\bar{k}_W - \bar{k}_B)
\]

where \( \bar{k}_{iB} = x_i^B \bar{k}_B + (1 - x_i^B) \bar{k}_W \). Let us again perform some comparative statics exercises. By differentiating this equation, we obtain:

\[
\frac{\partial y_i^*}{\partial z_iB} = \beta_1^{1/(2-\beta_1)}k_{iB}^{1/(2-\beta_1)}(\bar{k}_W - \bar{k}_B) \frac{\partial x_i^B}{\partial z_iB} < 0
\]

(46)

\[
\frac{\partial y_i^*}{\partial k_{iB}} = \beta_1^{1/(2-\beta_1)}k_{iB}^{-1/(2-\beta_1)}(\bar{k}_W - \bar{k}_B) \frac{\partial x_i^B}{\partial k_{iB}} > 0
\]

(47)

\[
\frac{\partial y_i^*}{\partial d} = \beta_1^{1/(2-\beta_1)}k_{iB}^{-1/(2-\beta_1)}(\bar{k}_W - \bar{k}_B) \frac{\partial x_i^B}{\partial d} > 0
\]

(48)

Let us now differentiate (46) to obtain:

\[
\frac{\partial^2 y_i^*}{\partial z_iB \partial k_{iB}} = - \beta_1^{1/(2-\beta_1)}k_{iB}^{-1/(2-\beta_1)}(\bar{k}_W - \bar{k}_B) \frac{\partial x_i^B}{\partial z_iB}
\]

\[
= - \beta_1^{1/(2-\beta_1)}k_{iB}^{-1/(2-\beta_1)}(\bar{k}_W - \bar{k}_B) \left[ k_{iB}^{(\beta_1-1)/(2-\beta_1)} - (\beta_1 - 1) (\bar{k}_W - \bar{k}_B) k_{iB}^{(\beta_1-2)/(2-\beta_1)} \frac{\partial x_i^B}{\partial k_{iB}} \right] \frac{\partial^2 x_i^B}{\partial z_iB \partial k_{iB}}
\]

The sign of \( \frac{\partial^2 y_i^*}{\partial z_iB \partial k_{iB}} \) is the same as the opposite sign of:

\[
\left[ k_{iB}^{(\beta_1-1)/(2-\beta_1)} - (\beta_1 - 1) (\bar{k}_W - \bar{k}_B) k_{iB}^{(\beta_1-2)/(2-\beta_1)} \frac{\partial x_i^B}{\partial k_{iB}} \right] \frac{\partial^2 x_i^B}{\partial z_iB \partial k_{iB}}
\]

and this is obviously indeterminate.
Appendix 2: Description of control and instrumental variables

**Individual socio-demographic variables**

*Female:* dummy variable taking value one if the respondent is female.

*Age:* respondents’ age measured in years.

*Health status:* response to the question "In the last month, how often did a health or emotional problem cause you to miss a day of school", coded as 0= never, 1=just a few times, 2=about once a week, 3=almost every day, 4=every day.

*School attendance:* number or years the respondent has been a student at the school.

*Student grade:* grade of student in the current year.

*Mathematics score:* score in mathematics at the most recent grading period, coded as 1= D or lower, 2= C, 3=B, 4=A.

*Organized social participation:* dummy taking value one if the respondent participate in any clubs, organizations, or teams at school in the school year.

*Motivation in education:* dummy taking value one if the respondent reports to try very hard to do his/her school work well, coded as 1=I never try at all, 2=I don’t try very hard, 3=I try hard enough, but not as hard as I could, 4=I try very hard to do my best.

*Self esteem:* response to the question: "Compared with other people your age, how intelligent are you", coded 1as = moderately below average, 2= slightly below average, 3= about average, 4= slightly above average, 5= moderately above average, 6= extremely above average.

*Physical development:* response to the question: "How advanced is your physical development compared to other boys your age", coded as 1= I look younger than most, 2= I look younger than some, 3= I look about average, 4= I look older than some, 5= I look older than most.

**Family background variables**

*Household size:* number of people living in the household.

*Public assistance:* dummy taking value one if either the father or the mother receives public assistance, such as welfare.

*Mother working:* dummy taking value one if the mother works for pay.

*Two married parent family:* dummy taking value one if the respondent lives in a household with two parents (both biological and non biological) that are married.

*Single parent family:* dummy taking value one if the respondent lives in a household with only one parents (both biological and non biological).

*Parents’ age:* mean value of the age of the parents (biological or non-biological) living with the child.

*Parent occupation dummies:* closest description of the job of (biological or non-biological) parent that is living with the child, coded as 6-category dummies (doesn’t work without being disables, the reference group, manager, professional or technical, office or sales worker, manual, military or
security, farm or fishery, retired, other). If both parents are in the household, the occupation of the father is considered.

**Protective factors**

*Parental care:* dummy taking value one if the respondent reports that the (biological or non-biological) parent that is living with her/him or at least one of the parents if both are in the household cares very much about her/him.

*Relationship with teachers:* dummy taking value one if the respondent reports to have trouble getting along with teachers at least about once a week, since the beginning of the school year.

*School attachment:* composite score of three items derived from the questions: "How much do you agree or disagree that a) you feel close to people at your school, b) you feel like you are part of your school, c) you are happy to be at your school", all coded as 1= strongly agree, 2= agree, 3= neither agree nor disagree, 4= disagree, 5= strongly disagree. (Crombach-alpha =0.75).

*Social exclusion:* response to the question: "How much do you feel that adults care about you, coded as 1= very much, 2= quite a bit, 3= somewhat, 4= very little, 5= not at all.

**Global social interaction variables**

*Friend attachment:* dummy taking value one if the respondent reports that he/she feels that his/her friends cares very much about him/her.

*Friend involvement:* response to the question: "During the past week, how many times did you just hang out with friends", coded as 0= not at all, 1=1 or 2 times, 2=3 or 4 times, 3=5 or more times.

*Friend contacts:* composite score of the values averaged on all nominated friends of the items derived from the questions: "Did you a) go to nominated friend (NF)'s house during the past seven days, b) meet NF after school to hang out or go somewhere during the past seven days, c) spend time with NF during the past weekend, d) talk to NF about a problem during the past seven days, e) talk to NF on the telephone during the past seven days, all coded as 1=yes, 0=no. (Crombach-alpha =0.84)

**Local social interactions variables**

*Strength of local interactions:* composite score of the values averaged on direct friends only of the items listed in the description of friend contact. (Crombach-alpha =0.86).

*Quality of local interactions:* average value across direct friends of mathematics score.

*Quality of local interactions (background):* average value across direct friends of parent education.

*Protective factors of local interactions:* composite scores of average value across direct friends of school attachment, parental care, friend attachment, social exclusion. (Crombach-alpha =0.72).

*Local average delinquency:* average value across direct friends of delinquency index

**Residential neighborhood variables**

*Neighborhood quality:* interviewer response to the question "How well kept are most of the
buildings on the street", coded as 1= very poorly kept (needs major repairs), 2= poorly kept (needs minor repairs), 3= fairly well kept (needs cosmetic work), 4= very well kept.

Residential building quality: interviewer response to the question "How well kept is the building in which the respondent lives", coded as 1= very poorly kept (needs major repairs), 2= poorly kept (needs minor repairs), 3= fairly well kept (needs cosmetic work), 4= very well kept.

Neighborhood safety: dummy variable taking value if the interviewer felt concerned for his/her safety when he/she went to the respondent’s home.

Neighborhood black population: percentage of black persons living in the respondent’s Census Tract.

Residential area type: interviewer’s description of the immediate area or street (one block, both sides) where the respondent lives, coded as 5-category dummies (rural, the reference group, suburban, urban - residential only, 3 or more commercial properties - mostly retail, 3 or more commercial properties - mostly wholesale or industrial, other).

School variables

Teachers’ quality: percentage of full-time classroom teachers holding Master’s degree of higher.

Teachers’ race: percentage of full-time classroom teachers that are black.

School quality: ratio between full-time classroom teachers and average class size.

School type dummies: catholic or other private with religious affiliation, private with no religious affiliation, public (reference group), other.

Students quality: number of students retained in the same grade for the next academic year (averaged on all grades and total amount of students held back if the school is ungraded).

Strictness of school anti-crime regulations: composite score of the items derived from the questions: "In your school, what happens to a student who is caught: a) injuring another student, b) possessing alcohol, c) possessing an illegal drug, d) possessing a weapon, e) drinking alcohol at school, f) using an illegal drug at school, g) verbally abusing a teacher, h) physically injuring a teacher, i) stealing school property", coded as 1= no policy, verbal warning or minor actions, 2= in-school suspension (the student does not attend classes, but comes to school), 3= out-of-school suspension (the student must stay out of school for a time), 4= expulsion (the student must withdraw permanently). (Crombach-alpha 0.74)

Instrumental variables

Interracial marriage: dummy taking value one if either the father or the mother is not of the same race of the respondent.

Religious affiliation: dummy taking value one if the respondent’s religion is African methodist, Holiness or Islam.

Religious practice: response to the question: "In the past 12 months, how often did you attend religious services", coded as 0= not applicable, 1=never, 2=less than once a month, 3=once a month or more, but less than once a week, 4=once a week or more.
Importance of religion: response to the question: "How important is religion to you", coded as 0= not important at all, 1=never, 2=fairly unimportant, 3=fairly important, 4= very important.

Intensity of faith: dummy variable taking value one if the respondent "agrees" to the question: "Do you agree or disagree that the sacred scriptures of your religion are the word of God and are completely without any mistakes?"

Fanatic faith: dummy taking value one if the respondent answers "yes" to the question: "Do you think of yourself as a Born-Again Christian?"

Religious youth group participation: response to the question: "Many churches, synagogues, and other places of worship have special activities for teenagers—such as youth groups, Bible classes, or choir. In the past 12 months, how often did you attend such youth activities?", coded as 0= not applicable, 1=never, 2=less than once a month, 3=once a month or more, but less than once a week, 4=once a week or more.
Figure 1: Returns to education from having white friends for blacks and whites
Figure 2. Distribution of adolescents by share of same race friends
Figure 3a: One Possible Interior Equilibrium for Black Teenagers
Figure 3b: One Possible Interior Equilibrium for White Teenagers
Figure 4: Example of a network with mutually consistent choices
Table 1: Race, academic achievement, friends and parental education

<table>
<thead>
<tr>
<th>Variable</th>
<th>Whites</th>
<th>Blacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index of school performance: ( y_{ij} )</td>
<td>0.65 (0.34)</td>
<td>0.36 (0.49)</td>
</tr>
<tr>
<td>Same race friends (shares in total number of friends): ( \bar{x}_{ij} )</td>
<td>0.84 (0.12)</td>
<td>0.58 (0.59)</td>
</tr>
<tr>
<td>Parental education*: ( k_{ij} )</td>
<td>0.61 (0.49)</td>
<td>0.32 (0.38)</td>
</tr>
<tr>
<td>Average parental education of friends: ( \bar{k}_{ij} )</td>
<td>0.62 (0.38)</td>
<td>0.30 (0.29)</td>
</tr>
<tr>
<td>Race (shares in total sample)</td>
<td>0.54 (0.51)</td>
<td>0.17 (0.39)</td>
</tr>
</tbody>
</table>

Notes:
- mean values and standard errors (in parentheses) are reported
- \( t \)-tests for differences in means are performed.
All differences of mean values are statistically significant at the 1% level.

*We use a dummy variable taking value of one if the parent is at least graduated from college and zero otherwise.

Table 2: Estimation results on key variables (Model 1)
Dependent variable: school performance index (n. obs. 10,003)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Whites</th>
<th>Blacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same race friends: ( \bar{x}_{ij} )</td>
<td>0.2501*** (0.0107)</td>
<td>-0.1109*** (0.0441)</td>
</tr>
<tr>
<td>Own parental education: ( k_{ij} )</td>
<td>0.1964*** (0.0094)</td>
<td>0.2832*** (0.0355)</td>
</tr>
<tr>
<td>Friends’ parental education: ( \bar{k}_{ij} )</td>
<td>0.1880*** (0.0075)</td>
<td>0.2445*** (0.0433)</td>
</tr>
<tr>
<td>Same race friends ( \times ) own parental education: ( \bar{x}<em>{ij} k</em>{ij} )</td>
<td>0.1231*** (0.0305)</td>
<td>-0.0181*** (0.0029)</td>
</tr>
</tbody>
</table>

\( R^2 \) | 0.6006 | 0.5561 |

Notes:
- standard errors in parentheses
- coefficients marked with one (two) [three] asterisks are significant at 10 (5) [1] percent level
Table 3: Descriptive statistics on instruments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Blacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inter-race marriage</td>
<td>0.19</td>
</tr>
<tr>
<td>Religious affiliation</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Religious practice</td>
<td>2.99</td>
</tr>
<tr>
<td>(0.99)</td>
<td></td>
</tr>
<tr>
<td>Importance of religion</td>
<td>3.02</td>
</tr>
<tr>
<td>(1.16)</td>
<td></td>
</tr>
<tr>
<td>Intensity of faith</td>
<td>0.15</td>
</tr>
<tr>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>Fanatic faith</td>
<td>0.21</td>
</tr>
<tr>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>Religious youth group participation</td>
<td>3.19</td>
</tr>
<tr>
<td>(0.94)</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
- mean values and standard errors (in parentheses) are reported

Table 4: First-stage estimation results on key variables
Dependent variable: percentage of black friends (only Blacks, n. obs. 3,591)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Blacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interracial marriage</td>
<td>-0.2457*** (0.0210)</td>
</tr>
<tr>
<td>Religious affiliation</td>
<td>0.3196*** (0.0109)</td>
</tr>
<tr>
<td>Religious practice</td>
<td>0.1598*** (0.0059)</td>
</tr>
<tr>
<td>Importance of religion</td>
<td>0.1131** (0.0605)</td>
</tr>
<tr>
<td>Intensity of faith</td>
<td>0.1595*** (0.0419)</td>
</tr>
<tr>
<td>Fanatic faith</td>
<td>0.2961*** (0.0498)</td>
</tr>
<tr>
<td>Religious youth group participation</td>
<td>0.1964*** (0.0094)</td>
</tr>
</tbody>
</table>

$R^2$ 0.6464

Notes:
- standard errors in parentheses
- coefficients marked with one (two) [three] asterisks are significant at 10 (5) [1] percent level
Table 5: Second-stage Estimation results on key variables
Dependent variable: school performance index (only Blacks, n. obs. 3,785)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Blacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same race friends: $\tilde{x}_{ij}$</td>
<td>$-0.1310^{***}$</td>
</tr>
<tr>
<td>Own parental education: $k_{ij}$</td>
<td>$0.2095^{***}$</td>
</tr>
<tr>
<td>Friends’ parental education: $\bar{k}_{ij}$</td>
<td>$0.2559^{***}$</td>
</tr>
<tr>
<td>Same race friends $\times$ own parental education: $\tilde{x}<em>{ij} k</em>{ij}$</td>
<td>$-0.01701^{***}$</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.5766</td>
</tr>
</tbody>
</table>

Notes:
- standard errors in parentheses
- coefficients marked with one (two) [three] asterisks are significant at 10 (5) [1] percent level